M2 Research Internship: Optimization approach to arbitrage-free models of price impact

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Subject

In order to model the trading cost due to a series of transactions, one often uses The Temporary Impact Model [1], which – in a discrete time setting – can be written as

$$p_t = \sum_{t'<t} \left[G(t-t') f(v_{t'}) \epsilon_{t'} + \eta_{t'} \right] + p_{-\infty}. \tag{1}$$

In this formula where p_t is the market mid-price just before transaction t, ϵ_t is the sign of the transaction (+1 for buyer, -1 for seller initiated) and v_t is the volume of the transaction. G is the so-called propagator describing the temporal evolution of impact, and f describes the dependence on the size v_t of the transaction. f is a general non-linear function, often a power-law of the form $f(q_t) = |q_t|^\delta$.

Gatheral [2] provides an excellent overview of such practical applications. In particular, under Eq. (1) the expected cost of a series of transactions can be calculated as

$$C[\{q_t\}] = \sum_{t=0}^{T} q_t \sum_{t'=0}^{t-1} f(q_{t'}) G(t-t'). \tag{2}$$

A trading strategy is considered a round-trip if

$$\sum_{t=0}^{T} q_t = 0. \tag{3}$$

A *price manipulation* or *dynamic arbitrage* strategy is defined as a round-trip with negative expected cost $C[\{q_t\}] < 0$. Gatheral [2] analytically derives several constraints on the functional forms of f and G to ensure the lack of dynamic arbitrage.

In earlier work [3] explored complex, neural network models that significantly outperform Eq. (1) in fitting the real price process. However, for such models no analytical or even empirical results exist regarding no-arbitrage conditions.

The property that an impact model admits no dynamic arbitrage is very important in practice. Automated optimizers of trading trajectories can easily pick up on – likely spurious – arbitrage and generate erratic trajectories in trading algorithms.

We propose to study specifically Eq. (1), and develop methods to validate the arbitrage relations known from [2]. These can be either based on finding a single optimal trajectory, or on a Generative Adversarial Network. The approach would then be used in two ways. First, we would generate arbitrage trajectories given a mis-specified model and look at their statistical properties. Second, we would apply it to remove arbitrage from models of varying complexity developed by Palmari [3].

References

[1] J.-P. Bouchaud, J. Kockelkoren, and M. Potters. Random walks, liquidity molasses and critical response in financial markets. *Quantitative Finance*, 6(2):115–123, 2006.

[2] J. Gatheral. No-dynamic-arbitrage and market impact. *Quantitative Finance*, 10(7):749–759, 2010.

[3] G. A. Palmari. *Interpretable non-linear models for price impact.* PhD thesis, Dauphine-PSL University, 2022.