M2 Research Internship: Optimization approach to arbitrage-free models of price impact

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Thesis possibility after internship: -

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Subject

In order to model the trading cost due to a series of transactions, one often uses The Temporary Impact Model [1], which – in a discrete time setting – can be written as

\[ p_t = \sum_{t' < t} [G(t - t')f(v_{t'}) \epsilon_{t'} + \eta_{t'}] + p_{-\infty}. \]  

(1)

In this formula where \( p_t \) is the market mid-price just before transaction \( t \), \( \epsilon_t \) is the sign of the transaction (+1 for buyer, −1 for seller initiated) and \( v_t \) is the volume of the transaction. \( G \) is the so-called propagator describing the temporal evolution of impact, and \( f \) describes the dependence on the size \( v_t \) of the transaction. \( f \) is a general non-linear function, often a power-law of the form \( f(q_t) = |q_t|^\delta \).

Gatheral [2] provides an excellent overview of such practical applications. In particular, under Eq. (1) the expected cost of a series of transactions can be calculated as

\[ C[\{q_t\}] = \sum_{t=0}^{T} q_t \sum_{t'=0}^{t-1} f(q_{t'})G(t - t'). \]  

(2)

A trading strategy is considered a round-trip if

\[ \sum_{t=0}^{T} q_t = 0. \]  

(3)

A price manipulation or dynamic arbitrage strategy is defined as a round-trip with negative expected cost \( C[\{q_t\}] < 0 \). Gatheral [2] analytically derives several constraints on the functional forms of \( f \) and \( G \) to ensure the lack of dynamic arbitrage.

In earlier work [3] explored complex, neural network models that significantly outperform Eq. (1) in fitting the real price process. However, for such models no analytical or even empirical results exist regarding no-arbitrage conditions.

The property that an impact model admits no dynamic arbitrage is very important in practice. Automated optimizers of trading trajectories can easily pick up on – likely spurious – arbitrage and generate erratic trajectories in trading algorithms.

We propose to study specifically Eq. (1), and develop methods to validate the arbitrage relations known from [2]. These can be either based on finding a single optimal trajectory, or on a Generative Adversarial Network. The approach would then be used in two ways. First, we would generate arbitrage trajectories given a mis-specified model and look at their statistical properties. Second, we would apply it to remove arbitrage from models of varying complexity developed by Palmari [3].

References

