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Crises propagation and behavioural effects in multi agents self-reflexive real business cycle models

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CRISES PROPAGATION AND BEHAVIOURAL EFFECTS IN MULTI AGENTS SELF-REFLEXIVE REAL BUSINESS CYCLE MODELS

FEDERICO GUGLIELMO MORELLI

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ABSTRACT

The Global Financial Crisis of 2008 left was also a crisis for macroeconomic models. On the one hand, orthodox economists persist in using the skeleton of classical models, while on the other hand, various groups of heterodoxes have proposed different ways to change the foundations of Economics. My research aims to bridge the gap between Neo-classical Economics and complexity Economics using methods and techniques from statistical physics. Beginning with standard economic models, I study the addition of a self-reflexive feedback impacting the confidence of individual economic agents. This induces large output swings despite only minor variations in economic conditions. Within this framework, economic crises propagate endogenously and are amplified by interactions. Later on, I enrich the previous framework by taking into account heterogeneities, studying how economic recessions propagate through different strata of society. In the last part of this work, I present a behavioural economic model where the stability of the economy is jeopardised by the lack of investments in risky markets.

PUBLICATIONS

This manuscript comprises the research work that the author (Federico Guglielmo Morelli) has conducted during the last three years under the supervision of Marco Tarzia, Michael Benzaquen and Jean-Philippe Bouchaud at the *Laboratoire de Physique Théorique de la Matière Condensée* (LPTMC) of Sorbonne Université and at the *Laboratoire d'Hydrodynamique* of the Ecole Polytechnique de Paris (LadHyX). The research have been conducted within the research *EconophysiX* chair, and includes the following published as well as original results.

Journal articles

- [1] Federico Guglielmo Morelli, Marco Tarzia, Michael Benzaquen and Jean-Philippe Bouchaud. " Confidence collapse in a multihousehold, self-reflexive DSGE model." In: *PNAS* 117.17 (2020), p.9244-9249.

Submitted articles

- [2] Federico Guglielmo Morelli, Michael Benzaquen, Jean-Philippe Bouchaud, Marco Tarzia. "Crisis Propagation in a Heterogeneous Self-Reflexive DSGE Model." In: *ongoing reviewing process PLOS One* (2021), arXiv preprint arXiv:2101.05588.

- [3] Federico Guglielmo Morelli, Karl Naumann-Woleske, Michael Benzaquen, Marco Tarzia, Jean-Philippe Bouchaud. "Economic Crises in a Model with Capital Scarcity and Self-Reflexive Confidence." In: *arXiv preprint arXiv:2109.09386* (2021).

CONTENTS

Part I

FOREWORD

FOREWORD

All truth passes through three stages. First, it is ridiculed. Second, it is violently opposed. Third, it is accepted as being self-evident.

— Arthur Schopenhauer

Usually, when I meet someone new revealing I am doing a PhD, I often find it difficult to describe my research to a layman. I am formed as a statistical physicist, but I later became interested in macroeconomic models.

Physics is mostly seen as the science that studies either subatomic particles at CERN in Geneva or stars and their gravitational fields.

Physics is the science studied in laboratories, possibly underground, away from other people. The results produced by physicists are not comprehensible to anyone except the members of the sect of physicists, who jealously guard them.

I went against this trend.

During this thesis, I chose to study a subject that is not typical among physicists. Indeed, economics is often seen as non-scientific and far removed from the "pure world" of natural phenomena. I was at the interface of two worlds, with many physicists thinking of my field as one that is purely motivated by cupidity and material interest, and with economists seeing me as an arrogant "know-it-all" hoping to reinvent a decades-old field in a few months

These comments may be clearly met with scepticism from the reader, as it may seem like a very harsh judgement, but I can attest to the reality of the various remarks different colleagues addressed to me during these years. Personally speaking, one of the things that has fascinated me the most during my PhD is the amount of common ground Economics and Physics share.

In particular, the branch of Physics that interests me is Statistical Physics, or rather the Physics of *Complex Systems*, i.e. the field of Physics which studies the emergence of complex macroscopic phenomena due to the interactions of its microscopic constituents. Complex Systems is a science very suitable for interdisciplinary applications, and has a particular flair for social sciences. Its main purpose is to study emerging phenomena in aggregate systems, where the study of the interactions between different elements is the fundamental key to the understanding of its phenomenology. From these interactions emerge surprising and deeply fascinating phenomena. The most classic examples are those of water and ferromagnetism. Physics aim to investigate how certain changes in macroscopic properties occur as temperature varies. How is it possible that when the system is heated, the water evaporates and the magnet loses its attractive properties? The explanation cannot be attributed to the individual components, but rather to the properties of the aggregate. The *modus operandi* of the statistical physicist is to create mathematical models, based on reasonable assumptions, that are able to reproduce observed natural phenomena. In Chapter 1, I will illustrate the details of those examples, emphasising on the modelling process and how, when one refines the assumptions, their outputs are more and more realistic.

By way of a mental exercise, in extreme simplification, the society in which we all live and move on a daily basis is composed of agents that, like water molecules, interact with each other. Each action, from the clothes we buy to the shares we invest in, is the result of interactions.

It seems therefore obvious that the work of the statistical physicist and the macroeconomist are compatible. However, there are radical differences between the two domains that should not be underestimated.

Undoubtedly, the nature of social interactions is more complex and articulated than the laws governing two adjacent water molecules. People often act impulsively, driven by feelings rather than reason. The fashion and art industries are two examples that we are confronted with every day. Although beauty should be a universal concept, our tastes are subject to the filter of our personality. This explains why the choice to buy a branded coat for three times the price of a more "standard" one, can hardly be considered a rational decision but rather a matter of feelings. On the other hand, when a physicist models water molecules, they do not pause to think about their decision-making process. As a result, the behaviour of vapour clouds is subject to the laws of thermodynamics and fluid dynamics, which are deterministic and well-established and understood. By combining these two contributions, physicists are able to forecast the evolution of weather conditions which, despite the control we have over models at the microscopic level, remain very imprecise due to the intrinsic chaotic nature of its dynamic evolution. Without a large amount of data, trying to make a macroeconomic forecast in the same way we predict the weather seems like an unattainable goal[1]. As we shall see in the course of this document, there are too many variables over which we have no control (through, for example, correct estimation) and, in some cases, we even ignore their existence. The spirit with which I have conducted my research aims to avoid this approach. To keep the similarity with climate alive, I would dare to write that my attitude is closer to climatology rather than to meteorology[2]. Climatology aims to provide probabilistic claims that certain phenomena will occur. No one would believe me if I claim that it

1 Maybe in 50 years it will be easier due to the increasing availability of data.

2 The metaphor is not accidental: some examples of studies connecting the domains of climatology and Economics exist in the literature, see Ref.[4–6]

will rain in Paris in exactly two years from now. Even with the most sophisticated software, weather forecasts are inaccurate within two days, let alone two years.

It would be quite different if I state that in two years it will rain in Paris, with a probability of 15%. This assertion is based on the chronology of seasonal rainfall in Paris, matched with the observed trend, and is correct in probabilistic terms.

My approach, as I will also discuss in the conclusion of this thesis, is focused on probability distributions rather than on trajectories [3] of the economy, which remain dominated by uncertainty.

To achieve results, however, models are needed.

The 2008 Global Financial Crisis (GFC) left a void in macroeconomic modelling. The existing ones proved themselves to be inadequate, as they failed to provide an answer to what had happened. Only after a few years, and after many adjustments, some were finally able to understand (and reproduce) what economists were looking for. This, as I will discuss in Ch. 2, created a deep rift in the research community which does not seem to have healed so far. The economic researchers are torn: the most conservative think the old axiomatic methodology is still valid, while others, revolutionary, believe that economic theories must be rebuilt from the foundations. The dialogue between the two communities, as the reader might guess, is sporadic and difficult.

The ultimate goal of my research is particularly ambitious, insofar as it wants to bridge the gap between these two communities. In the conclusions of this manuscript, I will summarise my point of view, trying to highlight what the next bricks to place are.

I hope that, in writing this thesis, I have managed to capture the key aspects of my work. My greatest efforts were directed towards the definition of the models I have developed during the thesis and the discussion of their solutions. The task is not easy, as the assumptions underlying the models have to be plausible, and their results must be in line with observations. The models I will present in the next chapters are *toy models*, see Ref.[7], designed only to illustrate a few stylised facts. Many ingredients are still missing for them to be considered mature. I strongly believe stylised facts are essential for understanding economic dynamics. Only once they are understood and mastered can one increase their complexity. During my work I have used different techniques. Wherever it was feasible to obtain analytical derivations, we exploited their power to shed a light on the results and develop economic intuition. When, instead, analytical techniques broke down on the hurdle of algorithmic complexity, we exploited numerical simulations. Through numerical simulations, we generated synthetic data to test our hypotheses. This thesis' struc-

3 Trajectories are traced and studied, but considered only as stylised facts. They mostly serve to form an intuition of the outcomes of the model.

ture is the following. In the next chapter, I will dig deeper into the main concepts of Complex Systems, with particular emphasis on two examples: (i) the Ising model shall convince the reader about the importance of networks and interactions and (ii) the modellisation process aimed at describing the phase portrait of water. Ch.2 focuses on the main criticisms that Dynamic Stochastic General Equilibrium models (DSGE) models have faced after the 2008 GFC. I introduce the baseline DSGE model and two extensions: (i) the addition of the capital market and (ii) the implementation of heterogeneities. Ch.3 is devoted to the modification of the benchmark DSGE model. I show how the implementation of simple features can effectively answer some of the criticisms that DSGE models are facing. In the following chapter, Ch.4 I present a first extension of the previous model where we account for income inequalities and social networks. Finally, in Ch. 5 I move a step towards Agents Based Models (ABM) model, yet without abandoning the DSGE philosophy, adding elements that were missing in the first two chapters, such as investors' preferences and capital market. In the conclusions, I summarise the main achievements of my PhD, giving my critical perspective. I discuss the limits of my approach and the missing parts that should be investigated in the future. Finally, I give my personal opinion on the debate.

I hope the reader will find this work interesting.

Part II

FROM COMPLEX SYSTEMS TO ECONOMICS

This first part has an introductory role. In its first chapter it is presented the methodology of the statistical physicist and it is explained why it should be relevant for the macro economic analysis. The second chapter, that concludes this first part, is devoted to the introduction of the macro economical models that will be studied in the next part of this thesis. The DSGE models are introduced in their simpler form and some of their extensions are presented because useful in the debate.

1 COMPLEX SYSTEMS AS AN INTERDISCIPLINARY SCIENCE

Roughly, by a complex system I mean one made up of a large number of parts that interact in a non-simple way. In such systems, the whole is more than the sum of the parts, not in an ultimate, metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole. In the face of complexity, an in-principle reductionist may be at the same time a pragmatic holist.

— **Herbert A. Simon** [8]

1.1 INTRODUCTION

The main subject of this dissertation is the application of concepts, methods and techniques of Statistical Physics to Economics and social sciences. More precisely, the field to which I have dedicated myself is that of Macroeconomics and, in particular, the models that are more popular among policy makers. As we will see in the next chapter, the 2008 Global Financial Crisis (GFC) has exposed major problems in the existing macroeconomic models, as they did not prove themselves capable of predicting the crisis nor of providing a solution to it. The great theoretical "void" left by the GFC has not been filled to this date. The community of experts does not fully agree on the path that best solves the problem. They can be divided into two macro-categories. The first and largest group believes that it suffices to modify the existing models to make them more plausible, while the second group of researchers, fewer in number, argue that the macroeconomic laws should be revisited in depth, questioning the axioms. This second group proposes a different methodology. The goal of my research is to try to understand (and implement) the main ingredients that bridge the gap between those two communities. In this chapter, I introduce some key concepts of Statistical Physics that may be instrumental to bridging this gap. In doing so I hope to convince you, the reader, of their efficacy in the domain of social science.

In physics, but perhaps it is more correct to say in epistemology, two opposite approaches of research have formed. The first one, defined as *reductionist*, holds that all natural phenomena can be understood starting from a fine understanding of the elementary constituents of matter. An example is the field of high-energy Physics, where research has focused on studying the atom and breaking it down into its fundamental components to understand the shape of nature. Through this process it has been understood that the atom is divisible and formed by the interaction of smaller particles, called protons, neutrons and electrons. The constituents of the atomic nucleus, are themselves divisible into even smaller particles called quarks, whose Physics is determined by a fundamental force called the strong nuclear force. By reducing the scale further, we enter the string theory domain. According to this formulation, every known particle would be a string that vibrates differently. Their scale is approximately the Planck scale 10^{-33} m and, given the impossibility for humankind to obtain such a resolution, the falsifiability of the theory is questioned. We will not dig deeper into the details of fundamental forces and string theory, as it is not the purpose of this introduction.

The second *modus operandi*, diametrically opposed to the first, is the *holistic* method. Within this paradigm, in a system made up of a multitude of interacting components, the laws governing the individual parts of the system do not allow for an easy deduction of the collective system, the behaviour of which needs to be studied as a single entity or aggregate. In particular, the properties of these aggregates themselves are extremely different from the original substance: the behaviours displayed are often surprising and unexpected. The physical properties of water, and the solid, liquid and gaseous phases in which it can be observed, cannot be inferred from the properties of quarks, nucleons or even those of a single H_2O molecule. To solve the puzzle, water has to be understood as an aggregate of interacting molecules. Another interesting example is Darwinism and the evolutionary theory. If we want to apply the reductionist method, living beings are made of cells, whose information is encoded in the DNA. The DNA is a structure made of nucleotides, each of which is composed by one of the four nucleobases (cytosine (C), guanine (G), adenine (A) or thymine (T)). To understand how different species have developed through millennia, one has to see the big picture. The chemistry of DNA alone does not allow one to understand the larger scale biological and ecological phenomena that have led to the species we can see today. Many other examples are present in nature. Listing all of them might result heavy for the reader. The core message of this paragraph can be summarised by the fact that sometimes the key to unveiling the rules of nature lies in the properties of the whole and not in the single components of the substance or, quoting P.W. Anderson: "*more is different*"[9].

1.2 MORE IS DIFFERENT

The domain of science called *Complex Systems* aims to understand how unexpected and sometimes surprising behaviours emerge at the aggregate level, or *collective behaviours*. The date that best corresponds to the founding of this domain of science is perhaps that of the publi-

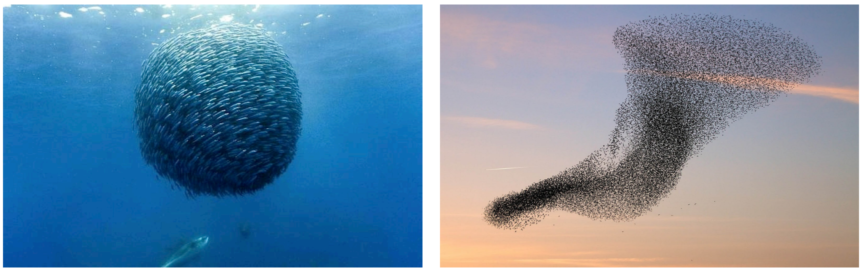

Figure 1: Two images depicting collective behaviour in the natural world. On the right, a school of fish arranges itself in a sphere to defend itself from possible predators, and on the left, a flock of birds swirls in the air, assuming harmonious shapes.

cation of "*More is Different*" [9] by P. W. Anderson, where, he claimed that:

"*Simple laws, rules, and mechanisms can, when applied to very large assemblages, lead to qualitatively new consequences.*".

Collective phenomena are the consequence of the interaction of the single elements of the aggregate. For example, no collective behaviour emerges from a heap of stones, as they don't interact. A heap of stones remains such.

To illustrate the striking properties of aggregates, nature provides a vast variety of examples of emerging collective behaviour. Two of the most fascinating and studied, [10–12] are represented by schools of fish and flocks of birds (unlike rocks, both fish and birds can interact with each other), as shown in Fig. 1.

The formation of a school of fish is a response to important external stimuli (such as the presence of food) that are transferred, via interactions, throughout the school of fish and creates coordination on a large scale. This coordination is also, for example, an effective defence mechanism against predators, increasing the chances of survival. Being in a group favours movements that disorient the predator and make the individual less identifiable.

The same reasoning applies, a few metres higher up, to flocks of birds. Another surprising and fascinating example of collective behaviour is the synchronization of fireflies, described very well by S.H. Strogatz and I. Stewart [13]

"*[...] along the tidal rivers of Malaysia, Thailand and New Guinea, where thousands of male fireflies gather in trees at night and flash on and off in unison in an attempt to attract the females that cruise overhead. When the males arrive at dusk, their flickerings are uncoordinated. As the night deepens, pockets of synchrony begin to emerge and grow. Eventually, whole trees* *pulsate in a silent, hypnotic concert that continues for hours."*

Similar to the case of birds and/or fish, synchronization is achieved via interactions. Each firefly sends and receives continuous signals from its neighbours, and it adapts its frequency accordingly.

These examples, just a few of many in the natural word, describe an emerging collective behaviour in which, in the words of Aristotle, "*the whole is greater than the sum of the parts*". What these systems have in common are interactions between individuals that act on a small scale (the distance between two birds, for example) and have large-scale consequences.

Intuitively, a substantial difference emerges when one observes a group of pigeons at rest in San Marco square in Venice or instead an equally large flock of swallows circling in the sky. Although the number of elements remains the same, the two systems are radically different. By an effort of imagination, we can speak of two different *phases*: uncoordination versus coordination. Statistical Physics is interested, among other things, in the study of the properties of these states and, in particular, in describing the underlying processes that lead to the transition from one state to the other. When this transition occurs, physicists call the process a *phase transition*. Phase transitions are one of the most striking and studied properties of aggregate systems made up of interacting parts. Nature offers a wide variety of examples of phase transitions, and in this chapter we will focus in particular on the properties of water and the difference between a magnetised and a non-magnetised metal.

The topics I will discuss later are set out clearly and extensively in the book *Statistical Mechanics: Entropy, Order Parameters, and Complexity* by J. Sethna [14][1], from which I took inspiration.

1.3 PHASE TRANSITIONS AND ORDER PARAMETERS

Experience teaches us that there are different *states* of matter in nature. With some of them we have daily contact. Three distinct phases of the same substance are the water we drink, the ice we use to cool our drinks, and the steam that forms in the shower. The same substance can have multiple phases and manifest itself in the form of each of them depending on certain factors. Intuitively, we know that ice, water and vapour are names we associate with different states of the same substance, water, which is composed of a multitude of H_2O molecules. If the temperature drops, water will turn into ice, but if we heat it long enough, it will evaporate. When the substance passes from one state to another, undergoing an abrupt change in physical properties – symmetry, for example – a phase transition occurs. The

1 I also suggest the reading of Yeomans' book "*Statistical mechanics of phase transitions*" [15] and *Equilibrium Statistical Physics* by Plischke [16]. See also Ref.[17].

processes of boiling as well as freezing are examples of phase transitions. Phase transitions are an intrinsic property of aggregates: in fact, the macroscopic properties of ice and water are very different from each other, but at the microscopic level both substances are formed by the same H_2O molecules.

Some questions arise spontaneously, and statistical physicists try to provide an answer: why do phase transitions exist? What are the factors that condition their existence? What is the value of these factors (e.g. why does water freeze at 0°C and not at 50°C)? To shed some light on the problem, we must proceed in order. It is necessary to establish a method that allows one to both rigorously describe the state of the system and to make predictions over its response to changing conditions. The first step is to isolate the key parameters that influence the physics of the problem. Physical systems have many degrees of freedom, and their role is not always evident. Many parameters will turn out to play an irrelevant role in the description of the phenomenology and will therefore be neglected. For this selection to be effective, it is necessary to systematically study their impact onto the system, by monitoring their effect individually. Those that will prove to be fundamental for the description of the aggregate system are called by physicists *control parameters*. Control parameters help physicists to orientate themselves in the different phases of matter and, depending on their value, to indicate its precise state. For an accurate description of the physical state of water, intuition suggests, temperature T, pressure P and volume V are key. If all three control parameters P, V and T are fixed, the substance appears in a precise state. A large enough change in temperature, as well as in volume or pressure, could induce a phase transition.

The second point concerns the identification of the phase of the aggregate system. To succeed, one must discard the possibility of controlling the behaviour of the individual elements of the group. This is too complicated and could prove to be misleading. For example, distinguishing between water and ice might be achieved by observing the periodic order of the H_2O molecules, see Fig.2 for a schematic representation of their different symmetries. Instead, as the arrangement of H_2O molecules is rather irregular, gas and liquid phases might be confused.

The identification of those aggregate observables allowing to quantify the phenomenology of the system is key. Such variables are called by physicists *order parameters* and their identification is neither a simple task, nor does it follow a standard procedure. Quoting Sethna: "*Choosing an order parameter is an art*". However, there are some general rules that a good order parameter must respect. From a mathematical point of view, it takes on different values in each of the phases of the system. The value of the order parameter is therefore a property of each phase, allowing physicists to characterise it unambiguously. Technically speaking, it should vanish in one phase, say phase A, and takes on non-zero values in phase B. To reconcile this difference, when a phase transition occurs, the control parameter has a singular behaviour. This discontinuity characterises the order of the phase transition. If the order parameter manifests a simple "jump" the phase transition is said to be *first-order*, otherwise the control parameter is continuous, and its singularity is encoded in the derivatives. The latter scenarios are classified by Physics as *second-order* (or higher) phase transitions. The values of the control parameters for which the order parameters manifest a singularity of any order are called critical points. The study of the critical point unveils the properties of phase transitions.

When studying the critical properties of water, physicists are particularly interested in the changes of its density ρ, as a function of temperature, the pressure and the volume of the *reservoir*. The density ρ, is a good order parameter as – in the case of many liquids – it changes considerably between two different phases, while it remains roughly constant within them. Intuitively, the density of vapour will be lower than both the density of water and ice, while ice remains less dense than water (ice floats)[2]. More specifically, the order parameter ρ undergoes a first order discontinuity when it reaches, at standard pressure for example, the critical temperature $T = T_c = 0°C$. To give some numbers, when $T = 0^{-}°C$ the density of the ice, $\rho_{ice} \approx 0.9 g/cm^3$ is considerably smaller than the density of water, which at $T = 0^{+}°C$ is $\rho_{water} \approx 1 g/cm^3$ [3].

Once the order parameter has been chosen, it is systematically studied in relation to each of the control parameters. The aim is to create a detailed map, noting all the positions of the critical points. The set of these points form lines that are defined as critical. This diagrammatic representation, usually two-dimensional for greater clarity, is called a *phase diagram*.

The leftmost panel of Fig.2 shows a schematic representation of the phase diagram of water as a function of the pressure and the temperature, keeping the volume constant. The lines shown in the figure are the collection of all the critical couples (P_c, T_c) for which one observes a phase transition (solid/liquid and liquid/gas).

Given a set of parameters, the phase diagram provides not only with precise information about the macroscopic properties of the system, but also with a measure of their sensitivity to parameters' variation. If the system lies well within the transition lines, then any reasonable variation of the control parameters will not have a radical effect on the system. *Vice versa*, if the system is close to the criticality, a variation, however small, of the parameters may induce the system

2 This is a peculiarity of water: for other substances, the solid phase is denser than the liquid phase.

3 We mentioned that the order parameter should vanish in one phase. This can be achieved by a simple redefinition of the density.

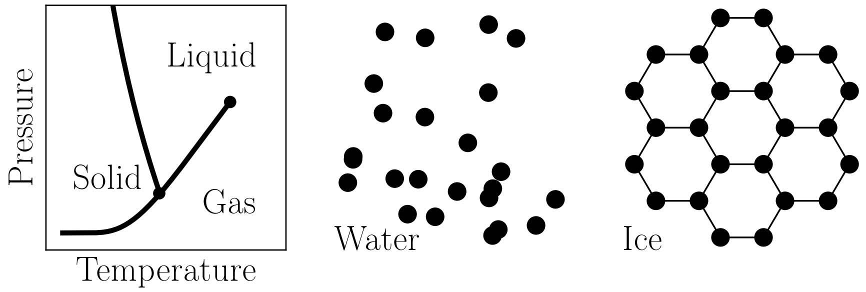

Figure 2: The rightmost panel of the figure shows a sketched version of the water phase diagram. The two rightmost panels instead compare the different symmetries of water and ice. Each dot corresponds to an O atom.

to a phase transition. To understand the extent to which phase transitions have a dramatic effect on the properties of the systems, it suffices to think that such an infinitesimal change, alone, is capable of altering simultaneously both the macroscopic qualities and the system's microscopic structure. One fundamental microscopic property that is spontaneously "broken" during a phase transition is symmetry.

For example, at normal pressure (1 atm) the liquid and solid phases of water are separated by an apparently negligible variation of the temperature (from 0^{+}°C to 0^{-}°C). This drop changes the macroscopic properties (liquid vs. solid) as well as the microscopic structure of the substance.

As illustrated in the two rightmost panels, water and ice have clearly different symmetries. The oxygen atoms in the solid and liquid forms of water form different patterns. On the one hand, the ice crystal possesses discrete rotational and translational symmetries. If one rotates the ice structure by a multiple of 60 degrees, the result will be indistinguishable from the original, and similarly, if one translates the ice lattice in a direction by an integer multiple of the basic lattice, the result will - again - be indistinguishable from the original. On the other hand, water is invariant in rotation and translation. In fact, any rotation or translation results in a representation that looks exactly like the original.

Regarding the importance of symmetries in Physics, P.W. Anderson wrote:

"It is only slightly overstating the case to say that Physics is the study of symmetry.".

By now, the reader should be convinced that aggregate systems exhibit unpredictable behaviours that, like phase transitions, cannot be deduced from individual components. At least two questions emerge and are inevitably interconnected. The first concerns a formal aspect of science. How is it possible to implement these collective phenomena in mathematically tractable theoretical models? The second question is more philosophical and focuses on the vast number of aggregate systems present in nature. What are the limits of validity of such models? In other words, can we somehow deduce the properties of different substances by developing a unified theory?

1.4 UNIVERSALITY CLASSES AND MODELISATION

The multitude of substances present in nature differs from each other both in their microscopical structure and in the way individual elements interact. At first glance, and because of the large number of possibilities, the role of physicists is extremely tough, as they necessarily need to study all those possibilities individually to infer their critical properties. Nature, however, simplifies the task as, regardless of the microscopical details, many systems are found to share the same critical behaviour. At the critical point, the correlations between the constituent elements of the substance become large scale [4], to the extent that the variations undergone by a single element affect elements far away from it. At the critical point, the collective effects completely dominate the phenomenology of the system, and the microscopic details become completely irrelevant in its description. The fact that the critical behaviour loses its dependence on the microscopic details, allows the mathematical description of its critical properties – i.e. occurring during a phase transition – to be surprisingly common to a wide variety of different systems. Those similarities are so recurrent that physicists started to classify different substances sharing the same critical properties into distinct classes, called *universality classes*. The study of critical behaviours then becomes the study of the universality class: as the study of one substance will provide "critical" information to all the others belonging to the same universality class.

One of the first clues of this universal behaviour was provided by E. A. Guggenheim in his work of 1945, see Ref. [18]. He showed how the liquid-gas critical lines (in the density[5] temperature plane) have the same mathematical properties for different atoms and small molecules, see Fig.3[6], that do not share the same microscopic properties. These striking similarities in the vicinity of a phase transition cannot be accidental and must be explained by a common theory. Nowadays, the distinct classes of universality are mostly identified and listed in various review works, see Ref.[19]. Despite the fact that, within the same universality class, critical behaviours have the same fundamental structure, the phase portraits of the different systems slightly differ from each other. Thus, to know the exact position of

4 In particular, the correlation length becomes infinite, and two constituent elements of the system become correlated regardless of their distance.

5 Density is the order parameter

6 Neon, Argon, Krypton, Xenon, N_2, O_2, CO and CH_4

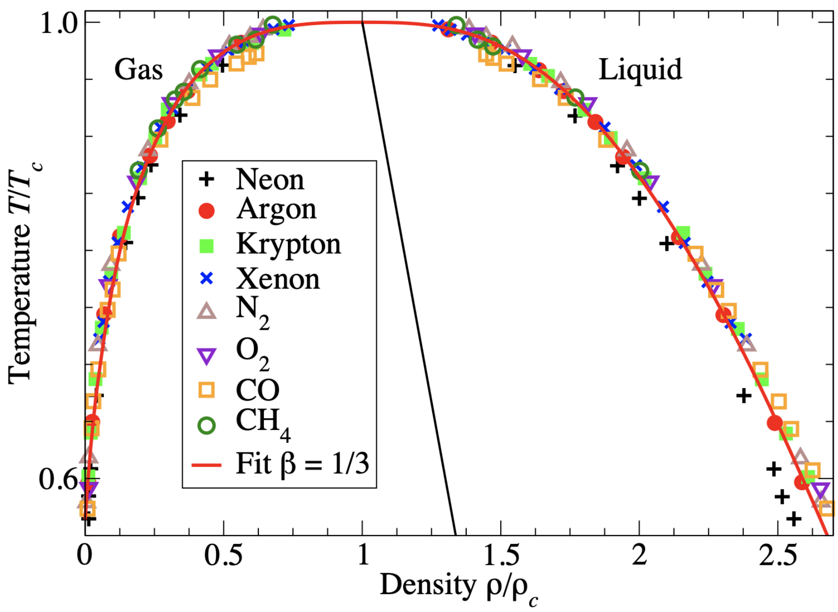

Figure 3: The figure shows how the critical liquid-gas coexistence lines superpose for different atoms and small molecules, near to the critical point. The picture is taken from the book *Statistical Mechanics: Entropy, Order Parameters, and Complexity* by J. Sethna [14] and it is based on the original work of E. A. Guggenheim [18].

the critical lines, it would be necessary to investigate each substance individually. However, if one is only interested in a qualitative study, the structure of the phase diagram remains very similar and, consequently, the underlying Physics. Carbon monoxide, CO, will display the same phases as water (solid, liquid, gas, ...) but the transitions occur at different values of the control parameters. At standard conditions, carbon monoxide manifests as a gas while water is liquid. Yet, surprisingly, critical properties are exactly the same for the two substances.

Physicists are therefore allowed to infer the physical properties of the molecule of dioxide from the properties of the carbon monoxide.

One of the most important consequences of universality classes, and it is also the reason why they have been introduced here, is the role they play in the process of modelisation. Thanks to universality, one can deduce the physical properties of very complicated substances by considering the simplest possible case. This will certainly be mathematically more tractable (intuition and understanding both benefit about this point) and at the same time it provides precise information about the phenomenology of the problem. For example, the Ising model (will be introduced later) is a perfect candidate to explore the paramagnetic/ferromagnetic transition, but it also provides the mathematical structure of all the phase transitions occurring within the same class of universality. To list some, the liquid/vapour transition of the water, binary mixtures transition, uniaxial magnetic systems belong to the same class as the Ising model, see [19] for additional references.

Surprisingly, the concept of universality is not restricted to Physics, and many examples of universal behaviour can also be found in Economics. Many distributions, including the distribution of incomes in different countries [20–22] and the growth rate of companies [22–25], are universal and described by the power-law distribution with similar exponents, despite all the countless differences that may exist at the level of the individual agent.

1.4.1 *The Process of Modellisation. The Case of Water*

Let's imagine one wants to describe the physics of water. Their goal is necessarily aimed to derive its phase portrait, and they are looking for the perfect model to achieve this goal. If they look closely, the water molecule is a rather complicated object. It is composed of two hydrogens forming two covalent bonds with an oxygen atom. As oxygen is more electronegative than hydrogen, it attracts more of the electrons involved in the chemical bond. The resulting electric charge is therefore not homogeneous but concentrated on two extremes, showing a dipole momentum. As an aggregate, dipoles interact via electromagnetic forces and keeping track of the resulting position of each molecule becomes really hard, as they are large in number. How is it possible to model all that? "It is not necessary" is the answer provided by the Complex Systems. One of the most important successes of the thermodynamics of the 19th century is the ideal gas law, formulated by Clapeyron, which provides a relation between pressure, volume, temperature and the number of particles for ideal gasses. The famous law reads as $PV = Nk_BT$, where P is the pressure, V the volume, T the temperature, N the number of particles and k_B a constant.

In the process of modelling, the assumptions are fundamental. An *ideal gas*, i.e. that follows the ideal gas law, has the following properties: (i) the molecules are so small in relation to the volume of the system that they are considered as dimensionless (punctiform); (ii) the gas is so diluted that all inter-molecular forces are neglected, and the particles only interact through perfectly elastic collisions. As a result, the ideal gas law is not able to provide any explanation for phase transitions. The reason relies on the fact that the underlying assumptions are too simple, and the interactions are mostly neglected, a part for the elastic collisions among particles. Although not satisfactory for our purpose, the ideal gas model is an excellent starting point. Two of the most natural extensions that can be made to the perfect gas formula are: (i) to exclude the volume occupied by the particles and (ii) to include the interactions among particles in the simplest possible way.

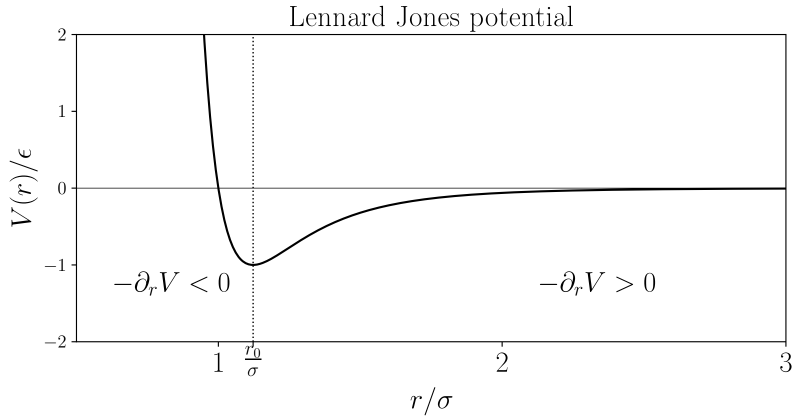

Figure 4: The figure shows the Lennard-Jones potential as reported in Eq.(2). The minimum of the potential is located at $r_0 = 2^{1/6}\sigma$. For $r < r_0$ interactions are repulsive ($-\partial_r V < 0$), while for $r > r_0$ the nature of interactions is attractive ($-\partial_r V > 0$).

First, to account for the impossibility of particles overlap the total volume is reduced by the volume occupied by the molecules, as $V \to V - Nb$, where b is the molecular volume of the single particle, and N their number. Intuitively, if the space occupied by the molecules Nb coincide with the size of the system V, the pressure becomes infinite. Second, the attractive interactions of molecules reduce the pressure: molecules close to the walls of the system are attracted towards the centre as they "see" a higher density. This translates into a reduction of the pressure by a term proportional to aN^2/V^2. Merging together those two ingredients lead to the so-called Van der Waals state equation, introduced in 1873 by the homonymous physicist [26], or:

$$P = \frac{Nk_BT}{V - Nb} - \frac{aN^2}{V^2} \,. \tag{1}$$

By introducing simple interaction effects, the Van der Waals equation of state predicts the transition between liquid and gas phases (but it misses the crystal phase), and it provides an estimation of the position of the critical lines.

The Van der Waals model still neglects many ingredients, and this omission is reflected by its imprecision. One of the defects of this formulation is that it only accounts for aggregate variables, *mean field* for a physicist. However, it is important to show how, even in such a simple approximation of the molecular interaction, one already grasps some key features of the system as one phase transition.

To date, it does not exist any state equation that provides a full explanation of the physics in all temperature ranges. If one wants to draw the complete phase diagram of water, it is necessary to abandon the mean field representation. The full phase portrait of water can be derived assuming particles interact via the Lennard-Jones potential

[27]. The Lennard-Jones potential of water molecules remain reduced to hard spheres (again, the dipole moment is completely neglected[7]) which, however, are subject to simple interactions. Molecules repel each other at short distances, while their interaction is attractive at long range. For the sake of clarity, Fig.4 shows the shape of the potential and distinguish the two attractive/repulsive regimes. The potential reads:

$$V(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] , \tag{2}$$

where r design the distance between two interacting particles, ϵ is the depth of the potential well and σ is the size of the particle (the potential vanishes for $r = \sigma$). It is therefore possible to associate to the system an energy, or *Hamiltonian*, $\mathcal{H}$:

$$\mathcal{H}(\{r_i, v_i\}) = \sum_j \sum_{i<j} V(|r_i - r_j|) + \sum_i \frac{1}{2} m v_i^2 , \tag{3}$$

where the set $\{r_i\}$ describes the particles' position, $\{v_i\}$ their velocities and m their mass (supposed to be the same). Indeed, the interaction potential V only depends on the relative distance between particles $|r_i - r_j|$. Statistical Physics teaches that every microscopic configuration, i.e. the collection of all the possible positions and velocities $\{r_i, v_i\}$, has a probability given by the Boltzmann law :

$$p(\{r_i, v_i\}) = \mathcal{Z}^{-1} e^{-\beta\mathcal{H}(\{r_i,v_i\})} , \qquad \mathcal{Z} = \int e^{-\beta\mathcal{H}(\{r_i,v_i\})} dr_i dv_i , \tag{4}$$

where $\mathcal{Z}$ is a normalization factor, also called *partition function*. The inverse temperature β is defined as $\beta := (\kappa_B T)^{-1}$ with T the temperature and $\kappa_B \approx 1.38 \times 10^{-23} J \cdot K^{-1}$ the Boltzmann's constant.

Combining the Hamiltonian described in Eq.(3) together with the Boltzmann probability, Eq.(4) one can fully reproduce the phase diagram of the water. Within this framework, the simplicity of the Van der Waals state equation is lost, but the correct critical behaviours can be found exploiting the renormalization technique [28] or, alternatively, they can be obtained through numerical simulations, e.g. through a direct integration of Newton's equations of motion or using a Monte Carlo algorithm, see Refs.[29, 30] for insights on the method and Ref.[31] for its applications to the Lennard-Jones potential.

This example aims to show how the process of refining hypotheses distorts the results. Starting from non-interacting hard spheres of the perfect gas law, a great improvement is achieved by taking into account the volume occupied by the molecules and the mutual attraction. The problem is only finally solved when the assumption of

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7 For this reason, the Lennard-Jones potential works particularly well with noble gasses.

treating the problem as a function of only global variables is relaxed and the molecules are modelled as individual entities.

In this approach, however, many details, such as dipole moments, are omitted. These details are essential if one wants to know the exact location of the phase transitions in the parameter space, but are not relevant as far as the qualitative shape of the phase diagram and the critical properties are concerned. The message the layman has to retain is that all the physics can be summed up in a simple interaction potential, such as the one described by Eq.(2). The lack of reductionist micro-foundations does not preclude the resolution of the problem, and indeed simplifies it.

A note that we will also recall later in this work: the Lennard-Jones framework increases considerably the complexity of the problem. The Van der Waals equation is easy to manipulate analytically, whereas in the case of the Lennard-Jones framework there is a price to be paid as one can only rely on numerical simulations.

1.4.2 *Interactions and Interdisciplinarity. The Ising Model*

What do magnets, voters in the United States, and the neurons in our brains have in common? At first glance, the answer is: "nothing". In reality, with a little imagination, all three systems share some properties and can, therefore, be described by the same model: the Ising model.

The Ising model is a perfect starting point for studying all those systems where binary variables are connected through a network. Magnets, as we shall see later, can be modelled as a set of spins lying on a lattice that can be in two states: "$\uparrow$" and "$\downarrow$". When the majority of the spins are coordinated and oriented along the same axis, the material is magnetic. Similarly, during the presidential elections, American voters can be represented with two binary values depending on whether they prefer Democrats, say they are in a state "1", or Republicans, corresponding to "-1". US citizens, even if they are not aware of it, form a network and their preferences are influenced, through constant interaction, by those of their neighbours. If the opinion of one agent differs completely from that of his neighbours, with a higher probability they will be persuaded (by the social pressure) to switch his preference.[8] When the majority of voters are synchronised, one party will be victorious over the second. Refs. [32, 33] applies the Ising model to vote-models. Neurons [34], similarly to magnets and US voters, can be switched on "$\circ$" or off "$\bullet$" depending on whether they receive an electrical impulse from their neighbours, which can only fire if they are themselves turned on. The water/steam transition can also be described by the Ising model: one can consider the

8 In some cases, they will change neighbourhood. Let assume this is not possible in order to keep the example simple

problem to be a lattice where nodes are either occupied or empty. A predominance of occupied sites represent the liquid state, while sparse occupation better describes the gas phase. The properties of the Ising model are worth to be discussed, as the model is very general and prone to interdisciplinarity [35]. To describe these systems correctly one would have to account for many details, making the task extremely complicated (in the case of magnets one would have to include dipole interactions, defects, etc.).

The concept of universality class, once again, comes to the rescue, and the research for a model is reduced to finding the simplest model that best represents the dynamics observed in nature. The Ising model accurately reproduces the qualitative aspect of most of those transitions, occurring between a phase where noise effects dominate (US citizens vote randomly) and a phase where one observes coordination between those binary variables.

The problem was first formalised in 1920 by German physicist Wilhelm Lenz and was designed to answer the question of why a magnet loses its attractive properties above a certain temperature. He imagined a magnet as a lattice made up of many small arrows, whose overall orientation explains the presence of magnetisation. The one dimensional case was solved by one of his students, Ernst Ising, from which the model take its name. The Ising model is formed by a collection, of binary spins, that can be represented as small arrows pointing into two possible directions: "↑" and "↓". Without loss of generality, one can associate to each spin the value 1 when it is in a "↑" state and -1 otherwise. The spins' interaction, say between first neighbours, determine their preference to align with other spins or to be oriented in the opposite direction (negative interaction). The *Hamiltonian* (energy) $\mathcal{H}$ of one configuration of N spins $\{s^i\}$, $i \in [0, N]$ can be written as:

$$\mathcal{H}(\{s^i\}) = -\sum_{i}^{N} \sum_{j \in \mathcal{N}(i)}^{N} J_{i,j} s^i s^j \ , \tag{5}$$

where $J_{i,j}$ represents the interaction matrix between spins i and j. $\mathcal{N}(i)$ is the collection of neighbours of the i-th spin. Thus, if we assume positive and homogeneous, anti-ferromagnetic, interactions among spins, $J_{i,j} > 0$ if $j \in \mathcal{N}(i)$ and $J_{i,j} = 0$ otherwise. Conversely, negative, ferromagnetic, interactions are accounted by simply assuming $J_{i,j} < 0$. For simplicity let assume $J_{i,j} > 0$ To each of the 2^N configurations, it is possible to associate a probability $p(\{s^i\})$, measured by the *Boltzmann's weight*:

$$p(\{s^i\}) = \mathcal{Z}^{-1} e^{-\beta \mathcal{H}(\{s^i\})} \ ; \qquad \mathcal{Z} = \sum_{\{s^j = \pm 1\}} e^{-\beta \mathcal{H}(\{s^i\})} \ , \tag{6}$$

where $\mathcal{H}$ is the Hamiltonian, Eq.(5), $\mathcal{Z}$ is the partition function and β the inverse temperature.

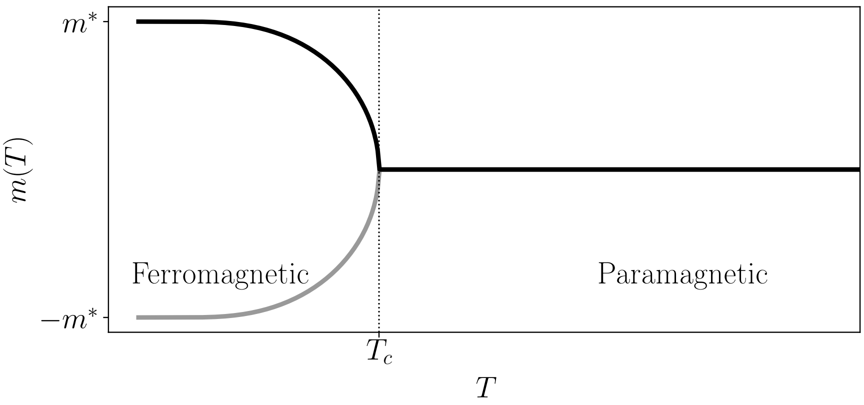

Figure 5: The left panel shows the global magnetisation m as a function of the temperature T. At $T = T_c$, m start being non-vanishing. The symmetry of the problem allow the spins to align in both directions (up and down) and the resulting average magnetisation is $m^* = 1$ and $-m^* = -1$ respectively.

From the definition of the Boltzmann's weight, one can deduce that it exists a competition of order versus entropy, corresponding respectively to two distinct limits. On the one hand when the temperature is very small $T \ll J$, the inverse temperature diverges, $\beta \to \infty$ and the measure is dominated by the states that minimise the Hamiltonian $\mathcal{H}$, Eq.(5). Thus, the system develops a spontaneous magnetisation, as most of the spins are aligned up or down. This limit is well represented by the leftmost panel of Fig.6, where a Monte Carlo simulation has been performed to 2D lattice[9]. The majority of the pins are aligned upwards, but for some isolated islands of "$\downarrow$" spins still persist (the temperature is not exactly zero). On the other hand, when the temperature is large $T \gg J$, the Boltzmann's weight of the (fewer) low-energy configurations is less important, the entropy effect dominates the statistical measure. In this limit the orientation of the spins is mostly random and the global magnetisation vanishes, as displayed by the rightmost panel of Fig.6.

It is reasonable, as it takes different values in the two limit (and it vanishes when $T \gg J$), that the average magnetisation is a perfect candidate as an order parameter to describe the paramagnetic/ferromagnetic transition. The average magnetisation $m(T)$, together with its thermal average, $\langle m(T) \rangle$ are defined as:

$$m(T) = \frac{1}{N}\sum_{i}^{N} s^i \,, \qquad \langle m(T) \rangle = \frac{1}{N\mathcal{Z}} \sum_{\{s^j\}=\pm 1}^{N} \sum_{i}^{N} s^i e^{-\beta \mathcal{H}(\{s^i\})} \,, \quad (7)$$

From the competition between entropy and order discussed above, the average magnetisation $\lim_{T\to 0} m(T) = \pm 1$, while it vanishes for

9 The solution of the 2D Ising model has been provided by Onsager in 1944 [36]

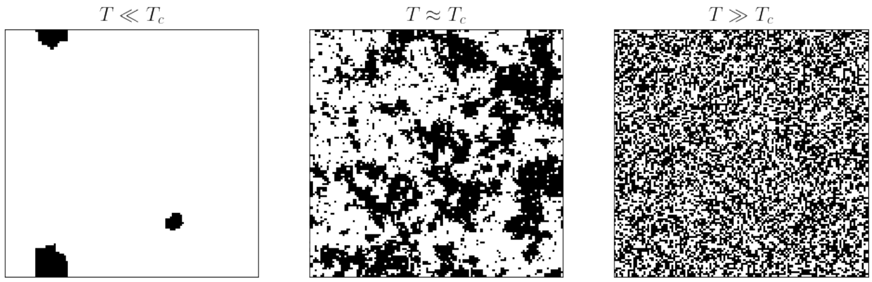

Figure 6: The figure shows the final configuration of a 2D Ising model with first neighbours interaction for different temperatures.From the left to the right. When $T \ll T_c$ the system displays a global magnetisation (the spins are aligned), at the critical temperature, $T \approx T_c$, clusters of different sizes of aligned spins coexist. Finally, when $T \gg T_c$ the entropic effects are dominant and the global magnetisation is zero. Simulations are performed using a Monte Carlo algorithm. The size of the lattice is 128x128 spins.

high temperature and $\lim_{T\to\infty} m(T) = 0$. For all other temperature ranges, the average magnetisation, $m(T)$, behaves as shown in Fig.5. The phase transition is spotted, and it is possible to clearly identify the critical temperature. For all temperatures above the critical value T_c, the effects of entropy are dominant and the magnetisation vanishes. Instead, when the system is cooled down and $T < T_c$ a spontaneous finite mean magnetisation emerges.

When one takes a closer look, the Hamiltonian described in Eq.(5) has a clear $\mathbb{Z}_2$ symmetry. In fact, the energy of a given configuration is exactly the same as the one having all the spins flipped ("$\uparrow$" to "$\downarrow$" and *vice versa*), as $\mathcal{H}(\{s^i\}) = \mathcal{H}(\{-s^i\})$. As a matter of fact, the probability density function, Eq. (6), shares this property.

An apparently counterintuitive results needs to be commented. When computing the thermal average magnetisation each of the terms $s^i e^{-\beta H(\{s^i\})}$ is odd under a $\mathbb{Z}_2$ symmetry transformation. Thus, each of their contribution sums to zero with the configuration having all the spins flipped. The resulting thermal average magnetisation, as written in Eq. (7) vanishes for each temperature T. So how is it possible that below a critical temperature T_c (Curie temperature) the system shows a finite magnetisation, say m^*? The only answer to this question is that the system (in the thermodynamic limit $N \to \infty$) has spontaneously broken the original $\mathbb{Z}_2$ symmetry. As in the case of water, the collective effects are so strong that they break the underlying symmetry the system possesses at the microscopic level. Once again, this phenomenon is an intrinsic property of aggregate systems and cannot be explained by looking at the isolated degrees of freedom.

What happens at the critical point? In Sec.1.4, it has been explained how, at the critical point $T = T_c$, the properties of the system no longer depend on the microscopic details. We saw how this effect is due to the divergence of the correlation length, which becomes infinite.

In the 2D Ising model, the correlation function $\mathcal{G}(r)$ is defined as:

$$\mathcal{G}(r) = \langle s^i \cdot s^{i+r} \rangle - \langle s^i \rangle \langle s^{i+r} \rangle \propto e^{-r/\xi} \ , \tag{8}$$

where $\xi \equiv \xi(T)$ represents the correlation length of the spins, see [36] for references. In physical terms, the correlation function provides a measure of how the orientation of the spin located at the site i affects the value of a spin distant r from it. Clearly, when $\mathcal{G}(r) \to 0$ the two spins are completely uncorrelated (this happens for $T \gg T_c$), while when $\mathcal{G}(r) \to 1$ the orientation of a spin i affects the sign of the spin placed at $i + r$.

At the critical point, $T = T_c$, the correlation length $\lim_{T \to T_c} \xi(T) \to \infty$ diverges and the correlation between spins becomes a power-law or long-ranged. Consequently, the response of the system to any, however small, perturbation is extremely strong and highly non-linear. Flipping a single spin, for instance, can switch many other spins, even very far away from it. Suddenly the network's structure loses its meaning, as each of the spins "sees" all the others. The macroscopic properties of the system have therefore detached from its microscopic details. This allows, as shown in the central panel of Fig.6, the formation of islands of aligned spins of all sizes. These islands behave like individual spins but on a larger scale. In the thermodynamic limit, their presence leads the system to be "scale invariant", or in simpler words, fractal.

However, it is important to remark that the exact value of the critical temperature T_c depends on the kind of microscopic interactions one considers, but the critical behaviours remain unchanged regardless of whether the microscopic parts of the system are iron atoms, people voting for the US elections or neurons, or whether interactions are modified to include more detailed and realistic features, e.g. dipolar interactions etc.

This simple, yet very rich, model provides a quantitative view of the paramagnetic/ferromagnetic phase transition. Let's sum up the achievements of this part. The concept of universality allows physicists to study very complicated phenomena just building models that only contains few key ingredients. Neither the Ising nor the Lennard-Jones models provide a detailed description of ferromagnetic materials and the water molecule, respectively, but yet they explain well the phenomenology of those problems. Despite their simple formulation, they are very robust to any change in the microscopical detail. In the case of the Ising model, the topology of the lattice and/or the range of interactions (nearest vs second-nearest neighbours) do not affect the final predictions of the model. In the case of the Lennard-Jones potential, one might think that the variation of the exponents (6-12) in Eq.(2), might alter the outcome of the theory but, in reality, any other potential that contains the same ingredients would lead to similar results.

What are the limits of those two theories?

1.5 THE ROLE OF DISORDER

A non-trivial role in phase transitions is played by *disorder*. Although its effects are not always predominant (as in the case of a diluted Ising model, i.e. some couplings are switched off $J_{ij} = 0$, see Refs.[37, 38]), disorder can have a dominant effect on the physics completely affecting the macroscopic phenomenology. The number of examples is large, and we will not dwell on them. This would deviate from the main purpose of this chapter, since models containing disorder are in general very complex to solve, and such a challenge would be difficult to introduce at this point. However, we will provide two examples that give the reader an idea of how the role of disorder can be dominant and that will come in handy later on. First, heterogeneities can be introduced associating to each spin allocated at site i an external random magnetic field h^i. The resulting Hamiltonian differs from the one introduced above, Eq.(5), and accounts for this modification as:

$$\mathcal{H}(\{s^i\},\{h^i\}) = -\sum_{i}^{N} \sum_{j \in \mathcal{N}(i)}^{N} J_{i,j} s^i s^j - \sum_{i}^{N} h^i s^i \ . \tag{9}$$

The resulting model is known under the name of "Random Field Ising model" (RFIM), see [39, 40]. The RFIM model still shows the ferromagnetic phase, but the criticality behaviour is completely dominated by disorder, allowing for hysteresis loops and avalanches. Going back to the example of American voters, this would translate into associating individual preferences with each person. The propensity of each individual to vote Republican, for example, will therefore be conditioned by its natural political orientation. Making him to vote against his natural tendencies would require a much bigger "persuasive effort" by its neighbours, generating a possible hysteresis cycle. Hysteresis simply states that the critical points are different whether one cools down or heat the system, see Ref.[41] for applications. In the case of neurons, this translates into an individual threshold each of the neurons has to achieve before turning "0", see Ref. [42]. Heterogeneities are particularly useful and interesting, and in the course of the Ch. 4 we will focus on a particular "disordered" extension of the model presented Ch.3. There I will present the effects that income heterogeneities have on the crises' propagation within the different strata of the society. The second way of implementing disorder, is to consider independent and identically distributed random pairings $J_{i,j}$ (dropping the assumption of first neighbours interactions). This modification completely disrupts the underlying physics and the system no longer shows the ferromagnetic phase. The resulting problem takes the name of *spin glasses*, and it is extensively studied in the literature, see Ref.[43].

These two examples give the reader an idea what are the boundaries of a model, i.e. under what assumptions its results are no longer valid.

1.6 FROM COMPLEX SYSTEMS TO ECONOMICS

After these premises, a fundamental question emerges: how does the job of a statistical physicist combine with Macroeconomics? How can the concepts of universality classes [10] and phase transitions apply to social sciences?

The consequences of non-linearities in the proximity of any phase transition have dramatic repercussions if not well understood. Both in the Ising model and in the case of water, next to the criticality, an infinitesimal change in a control parameter (e.g. temperature) leads to a macroscopic change in the state of matter: coordination vs uncoordination and liquid vs solid respectively. Intuitively, if one observes water at 30^{-}°C or 30^{+}°C they would hardly notice any difference, even disposing of a thermometer. *Vice versa*, our observer will not be puzzled between two samples at 0^{-}°C and 0^{+}°C. Thus, in a complex system the presence of a phase transition does not allow one to freely change the value of the control parameter, instead requires some degree of prudence. Alternatively, one has to accept its consequences.

Let's transpose this example to Economics. Imagine a policy maker which goal is to decrease the interest rates (temperature) while adopting other policies to keep inflation constant (pressure). If, for whatever reason, the economy was at the edge of a *tipping point,* any minor drop of the interest rates at constant inflation would have dramatic consequences on the macro level. Admittedly, comparing the economy to water is reductive to say the least. On the one hand, one might observe that the phase diagram of water is nowadays largely understood and drawn with extreme precision, and the underlying physical laws are known. On the other hand, the laws that determine people's behaviour and, therefore, the performance of the economy are, and will remain to a large extent, unknown. *A priori*, it is even challenging to claim that a phase portrait of economy exists. However, the importance of drawing a good phase portrait is key. In the decision-making process, the policy makers have a valuable tool to estimate the risk the economy incurs but, also, it provides them with an insight on how to overcome possible phase transitions or, in other words, economic crises. Monitoring the order parameter, they will realise whether they are on the edge of a tipping point and will be able to reverse the trend by acting directly, through policies and narratives, on the control parameter.

10 Even if we are not sure that such concept holds in Economics

As a parenthesis: much has been done in the past years by economists and many problems have been already thought through. The goal of my research, is to inject new ideas into the debate. Those concepts are borrowed from Statistical Physics and are mostly not yet applied to the economic research.

In this document I propose a Statistical Physics approach to modify some classical macro economical models. The lesson of P. W. Anderson teaches that we cannot ignore the fact that society (and therefore its economy) it is composed by a multitude of interacting agents. Similarly to the spins in a magnet, the aggregate of agents can show behaviours that are not explained when one focuses on one single representative of the population. Many dynamical concepts, such as imitation processes, crisis propagation, feedback loops and panic effects (to cite a few of them) can only be modelled in an environment where the population is seen as an aggregate of interacting components. Although the inclusion of social networks and interactions allow for a more realistic description, their nature is admittedly very intricate and, to some extent, not fully understood. In that regard, one needs to remember that universality comes to rescue. In fact, in this chapter I discussed how, many binary systems can be mapped onto an Ising model, whose properties are known. Universality allows reducing the microscopic description to a minimum, without losing information.

In Physics, the intellectual process of modelling strips the system of its frills and seeks to retain only its key aspects.

This aspect is caught in Sec.1.4.1 where I presented the Lennard-Jones potential. Within this framework, even barring few (not minors!) details of the water molecule, the results are still consistent with the experience.

This is the spirit with whom I conducted my research. In the models that are proposed in this thesis, both the agents and the interactions will be simplified as much as possible. There are two reasons for this: (i) the concept of universality teaches that even with a very simple framework it is still possible to qualitatively describe complex scenarios[11] and (ii) simplifying allows reducing the number of degrees of freedom. Reducing the number of parameters favours a better development of both the intuition and the understanding of the model. If, however simple, a model is built on a reasonable basis, it will have the same role in the macroeconomic investigation as the representative of a universality class. It can serve as a directive for policy makers to identify what are (i) the control parameters driving the economy and (ii) what are the important indicators of the economy, i.e. the order parameters.

11 Once again, one should think about how simple the Ising model is and how sophisticated its results are.

For each model I will present, we will systematically explore the parameters' space[12] to understand which are the important directions. The possible regimes described by the economy are explored, starting from their pictorial representation in a phase diagram (which also unveils their existence). In the course of the chapters, we will discuss many of the concepts introduced here in more detail, also providing concrete macro economical applications that will help the reader to form an intuition. In the next chapter, I will present the benchmark monetary models I have been working on during my thesis.

From Complex Systems to Economics: Highlights

This introductory chapter plays the role of a methodological manifesto. It outlines the key concepts of Statistical Physics with a focus on possible applications in Macroeconomics. In particular, it presents key concepts such as aggregate systems, control and order parameters, phase diagrams and universality. In order to provide the reader with suitable examples, two in particular are introduced: the Ising model, which describes the paramagnetic/ferromagnetic transition, and the Lennard-Jones model, which allows the phase diagram of water to be drawn accurately. The last part is devoted to a discussion of how these concepts find their way into the macroeconomic debate. More generally, this chapter provides an overview of the Econophysics approach to Macroeconomics and justifies the methodology that will be used in the following chapters.

12 The set of all parameters.

2 FROM CLASSICS TO NEW APPROACHES: AN OVERVIEW OF MACROECONOMIC MODELS

If farming were to be organised like the stock market, a farmer would sell his farm in the morning when it was raining, only to buy it back in the afternoon when the sun came out.

— John Maynard Keynes

In the previous chapter, we focused on introducing some key concepts underlying complex systems. The class of economic models I studied during my PhD is the dynamic stochastic general equilibrium (DSGE) models. Before presenting them, I would like to spend a few words introducing the intellectual path that led economists to their formulation. The aim of this first section is to guide the reader through the tipping moments in modern economic history that have led economists to reflect on the foundations of the benchmark models.

2.1 A BRIEF INTRODUCTION TO MACROECONOMIC MODELS

To paraphrase A. Kirman "*The economic crisis is a crisis for economic theory*" [44], the history of economic models is rooted in the history of their failures [45]. Very similarly, the most famous theories of Physics were born from the ashes of the previous ones. In this spirit, I will briefly review the major economic crises of the last century, trying to understand how macroeconomic models have adapted to overcome them. The three biggest economical crises of the last century can be traced back to the Great Depression (GD) of 1929, see Fig.7, the Great Inflation (GI) that swept the world from the mid-60s to the early 80s and finally the more recent subprime crisis, that led to the Global Financial Crisis (GFC) of 2008. The presentation of events is partly inspired by the article of Vines and Wills [46] "*The rebuilding Macroeconomics theory project: an analytical assessment*", written in the framework of the project "*Rebuilding Macroeconomics*". On some issues, and in particular *circa* the "revolution" of Keynes, I have juxtaposed Minsky's perspective as it appears in his book "*John Maynard Keynes*"[47].

2.1.1 *The 1929 Great Depression*

Before the Great Depression, the macroeconomic landscape was dictated by the Alfred Marshall (1842-1924) school of thought. Their goal was to understand what was the process underlying markets (the labour market, the goods market) equilibrium, namely the match between supply and demand sides. The Marshall's economic view was based on the concept of *ceteris paribus*[1]. The markets' equilibrium conditions are studied by assuming them as uncorrelated entities, i.e. without considering the influence of other markets. This approach is generally referred in the literature as *partial equilibrium*. The Great Depression fostered fundamental contributions to the construction of modern economic models. The mainstream economists believed in the self-correcting properties of markets, through an *invisible hand*[2] and their strategy was "wait-and-see". The lack of useful active economic public policies from these models during the GD set the stage for John Maynard Keynes' (1883-1946) ideas to have a strong impact in the field of Macroeconomics, studying the equilibrating tendencies of markets with mutual feedbacks. After the stock market crash of October 29th, 1929, the US economy experienced a recession of enormous magnitude. To give an idea of the amplitude of the crisis: the Down Jones market index, see Fig.7 dropped by almost 50% in two months (but more than 20% in the first day), and unemployment increased from 0.4% of August 1929 to 25.6% of May 1932[3]. Addressing the problem of unemployment for someone trained with the Marshallian method was relatively simple. In Marshallian terms, unemployment is merely a consequence of firms keeping wages above the market level. The mismatch between the demand and supply of labour can be solved by cutting down wages. At lower wages labour supply drops (working is less attractive), while demand increases, as it is profitable to hire. This double effect allows matching the labour's demand with the supply and, even better, a wage cut is able to restore full employment[4]. In his "*General theory*" [49], Keynes observed that if wages are cut, but there is no simultaneous raise in the aggregate demand for goods, then firms are not able to increase their production and consequently the labour demand. According to the Marshallian view, wages are the result of interactions between industry and employees, with central banks being an external actor to the process. Similarly to the wage/labour's demand relation, another obvious problem for partial equilibrium theories was the savings/investments relation and the role of interest rates. When interest rates are high, the demand for investments decreases as the cost of money increases (borrowing becomes more expensive). In this situation, not only firms do not ask for loans, but also people are also more eager to save. In fact, savings are promoted by the propensity of households to invest as the returns are high. Marshallian economists thought that the cut of interest rates was enough to increase the investments' demand, matching it with savings' supply. As the regulation of the inter-

1 From Latin: with all other condition being fixed
2 Concept introduced by Adam Smith (1723-1790)
3 Data provided by the National Bureau of Economic Research [48]
4 The unemployment level sustainable for the country.

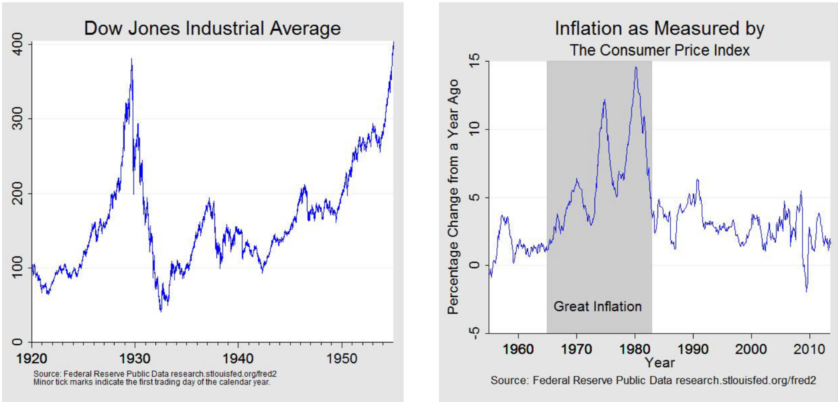

Figure 7: The graph illustrate the two main economic crises of the 20th century. The left panel shows the Dow Jones Industrial Average (market index) between 1920 and 1956. In the figure, it is clearly visible the 1929 stock market crash. The recovery only occurred in 1955, where the industrial average went back to comparable levels pre-crisis. The right panel shows the percentage variation of inflation from 1955 to 2015 approximately. Highlighted in grey is the period known as the Great Inflation.

est rates is uncorrelated from the other sectors (partial equilibrium), lowering the interest rates has no consequences on demand. Within this framework, it is impossible that an excess of savings lead to a fall in the production. In opposition to this paradigm, Keynes observed that interest rates must be determined accounting for *liquidity preferences* of investors. According to the theory of liquidity preferences, the interest rates must compensate the duration and the risk underlying investments, otherwise investors will prefer liquid holdings, e.g. cash. The interest rates should therefore be adjusted to ensure the match of demand and supply of liquidity. In Ch.5 I present a model where we account for a mechanism very similar to liquidity preferences theorised by Keynes. Agents invest in a portfolio according to a Sharpe ratio: if interest rates are more appealing than the returns the firm proposes, they will stop supplying capital for the production. Consequently, the output will diminish and recessions are more likely to occur.

According to Keynes the problem of equilibrium was misleading *per se* as it is deeply bonded with capitalist economies. In a capitalist economy a full-employment state is at best transitory, and it is intrinsically subject to successive booms and busts. His revolution (which was not entirely accomplished [47]) concerned observations on the quantity of money. The shortcomings of capitalism were due to its necessity of a financial market. Here *animal spirits*[5] are transformed into effective demand for investments [47], making the structure of financial system intrinsically fragile. In economic terms money and

5 A concept nowadays central into the macroeconomical debate

finance are non-neutral[6]. On the other hand, money neutrality was one of the cornerstones of classical economists. For a Marshallian economists, prices would adjust (prices are flexible) to ensure the market clearing, i.e. the match of the demand with the supply. For Keynes, on the other hand, if central banks increase inflation by injecting money into the market, the resulting inflation would not raise nominal wages, leaving their real value unchanged. In this perspective, crises, i.e. low output and unemployment, were endogenously driven via feedback effects and inherent to the system. As a parenthesis: following Minsky[47] the endogenous nature of economic recessions, nowadays central to the debate, was never taken too seriously by the orthodox economists who set it aside. If wages don't readjust (sticky nominal wages) according to the variation of prices, the market clearing cannot be achieved. To avoid recessions it is therefore necessary for the state to actively intervene with targeted investments to ensure full employment and the movement of goods. These concepts profoundly challenged the mainstream idea of self-regulating markets via the help of an invisible hand. The invisible hand replaced the role of Central banks, as there was no need for government interventions. The change of methodology in Macroeconomics, after Keynes, should have been radical.

According to the classic Economics interpretation [46], some efforts were still necessary, Meade [50–52] and Hicks[53] among others, to understand the general equilibrium properties of the General Theory. According to Vines and Wills [46] through the work of Keynes, a transition has occurred both in the content of economic theory (with the introduction of nominal wage rigidity, consumption function, liquidity preferences etc.) and conceptually with a move to the general equilibrium[7] (GE). According to a more critical point of view, see Minsky [47], the implementation of Keynes' ideas into the "New Keynesian" school is only partial. Many key concepts that Keynes wrote about were deliberately ignored. In particular, if the effort had been directed into implementing two key concepts as (i) animal spirits as a driver of financial decision-making process and (ii) non-equilibrating markets, the current debate would be different. According to Minsky [47], Keynes' revolution did not take place for two reasons. First, he could not support his ideas, as he died only 10 years after the publication of the General Theory. Secondly, the almost thirty years of economic growth (40s-60s) misled the economists. In his book, "*John*

6 If money was neutral they would only affect nominal values, but in real terms (quantities re-scaled by the price) nothing would change.

7 The concept of "General Equilibrium" was first introduced by the French economist Walras [54], who stated that markets find a General Equilibrium through the progressive adjustment of both demand and offer sides. This process is called *tâtonemment*. The properties of Walrasian general equilibrium are still central in the debate, see Ref.[55, 56]

Maynard Keynes" Minsky states:

> "*As the sixties progressed, eminent economists—especially those associated with government policy formulation who in their own minds were disciples of Keynes, were announcing that endogenous business cycles and domestic financial crises were a thing of the past, now that the secrets of economic policy had been unlocked. [...] In Economics if history over a thirty-year period does not cast up observations with at least a family resemblance to a financial panic or a deep depression, then arguments to the effect that these anomalies are myths, or that what happened can be explained by measurement errors, human (policy) errors, or transitory institutional flaws which have since been corrected, may be put forth and gain acceptance.* "

The post WWII models are often referred to as "New Keynesian" although many question the misuse of the Keynes reference, see Minsky[47].

The Marshall Plan, which was implemented on a global scale to lift countries' economies out of the disasters caused by the Second World War, which consisted in massive public investments meant to achieve full employment, was largely inspired by Keynesian theories.

2.1.2 *The 1970s Great Inflation*

Classic ideas dominated economic policies from the end of World War II up to the 1970s, when a period of Great Inflation (GI) occurred. During the GI, many advanced economies suffered a period of great economical stagnation and, at the same time, high inflation (12% on average in the decade 1970–1980 [8]). When *stagflation* [57] (combination between the words stagnation and inflation) occurred, policy makers were unable to adjust and control the inflation, that was further increased by the raise of crude oil price [9]. Macroeconomical theories were, once again, challenged as they were unable to explain the origin of stagflation. Following the Vines and Wills analysis, two schools of thought emerged. The "evolutionists" wanted to keep the classical framework, but enriching it with few mechanisms that would have been able to explain stagflation. In the second group, more radical than the first one, were instead the "interventionists"[10], predicting a

8 Data retrieved from *FRED economic Data* website, `https://fred.stlouisfed.org/series/FPCPITOTLZGUSA`

9 A first significant raise of the oil price occurred in 1973 due to the Arab Oil embargo (5 months), followed by a second crisis, in 1979 during the Iranian revolution. Overall, in the decade from 1970 to 1980 the crude oil price raised by more than twenty times, from 1.8$/barrel of 1970 to the 36.8$/barrel of 1980. Data are retrieved from the website Our World in Data and are available for consultations. The raise in crude oil price made energy, and therefore production, more expensive and less competitive.

10 Vines and Wills name this second group the "revolutionaries". The term seems too strong, as both groups fundamentally agreed on the nature the macroeconomical

complete change in the paradigm. I want to stress the formation of two distinct groups as it has many analogies with today's split between DSGE and ABM schools; even though today the gap is much deeper and radical. In the following, we will mainly focus on the debate over those aspects that will have a role in the next chapters, see Wren-Lewis for more details [58]. Two key concepts were introduced by the evolutionary school: (i) *adaptive inflation expectations* [59, 60] and (ii) the *nominal price anchors* (see the discussion about the flexibility of prices made above). The justification for the introduction of an adaptive inflation's expectations is simple. The persistent increase in the aggregate demand would raise over time, not only the inflation itself, but also the expectations over its growth.

One successful way of introducing a nominal anchor controlling the inflation was proposed some years later by Taylor, who gives its name to the homonyms rule [61].

The Taylor rule is a prescriptive tool for monetary policy for restoring equilibrium [11] as it gives the reaction that the Central Bank's real interest rate should have confronted to inflation variations and/or to unemployment. It provides a simple relation between the interest rates, the targeted inflation (acting as a price anchor) and the output gap, the difference between the real output and the potential output of the economy, i.e. the output at its full employment. According to the Taylor rule, when inflation grows above the target, say by 1%, the nominal interest rates (e.g. the federal funds rate) should raise by more than 1% so that the real interest rates (interest rates minus the inflation) are positive. A general form of the Taylor rule reads:

$$r_t = r + \pi_t + a_\pi(\pi_t - \pi) + a_y \left(\frac{Y_t - Y}{Y} \right) , \tag{10}$$

where r_t is the nominal interest rates, π_t and π the current and the target inflation[12] respectively, $(Y_t - Y)/Y$ a measure of the output gap, with Y the output level at full employment, and r the interest rates at equilibrium. a_π and a_y are two coefficients that Taylor fixed to 0.5 [61].

This should slow down the economy when the inflation grows, anchoring prices. Similarly, when the output levels are above the full employment (ideal level of output) the increase in the inflation cools down the economy.

The Taylor rule proved itself to be quite a powerful tool to control stagflation, and it took a central role in the macroeconomic analysis.

models should have. In the current debate, I would dare to call the ABM supporters "revolutionaries" as they insist for entirely new foundations.

11 Although it has been shown that even if the Taylor rule stabilises inflation dynamics it is not a sufficient condition for the economy to converge to equilibrium if many exist [62]

12 It has been proven recently that the Taylor rule does not ensure convergence to the inflation target, which is driven by self-organising effects of agents, see [63]

The role of the Central Banks also changed: from actively pursuing the full employment via public investments (as occurred with the Marshall plan) to actively anchoring the price levels via the interest rates [46].

The innovations brought by the interventionist group also deserve some attention. In this document, I will mainly focus on the famous *Lucas critique* [64], where the author questioned the forecasting powers of the existing models. In his article, Lucas argued that all the decisions made by policy makers were intrinsically wrong as the calibration of macroeconomic models, and therefore economic policies, was achieved exploiting pre-existing data. Policies are taken *as if* sectors were not responsive to them, while in reality the economy adapts to policies' changes. The initial forecast based on *prior* data would therefore prove itself to be outdated. The predictive power of macroeconomic models was challenged.

What Lucas was implicitly questioning is the ergodicity of Economics[65]. Following Davidson *"[...] Ergodicity implies that future outcomes are merely the statistical shadow of past and current market signals."* [66]. Samuelson wrote in 1969 [67] that if economists wants to move Economics from "the realm of history" into "the realm of science" then ergodicity must be imposed. The question of Economics being ergodic is very delicate, and I will comment more on that in the conclusions of this document.

In order to avoid the Lucas' paradox , they introduced the concept of *rational expectations*. Today, we know that rational expectations (the whole concept of rationality more in general) are one of the most controversial points of the DSGE models [68, 69]. To coherently implement rational expectations, Lucas and Sargent [70] advocated the need for rational agents to optimise their utility function accordingly. To ensure rationality, the macroeconomic agent is assumed to know the precise state of the economy and its functional form. This infinite knowledge allows the representative agent to maximise the stream of expected utility according to the future expectations over the new policies. An interesting justification for the introduction of rational expectations was provided by B.McCallum [71] where he commented:

"There are, I believe, two main justifications for this view. First, there is no reason to believe that the assumption is terribly inaccurate, empirically, at the macroeconomic level. Of course, it is literally untrue, but so is every behavioural relation in every formal economic model. Second, every alternative assumption has an extremely unattractive property: it requires the assumed existence of some particular pattern of systematic expectational error. "

The representative agent becomes a fully rational, forward-looking, maximiser of utility.

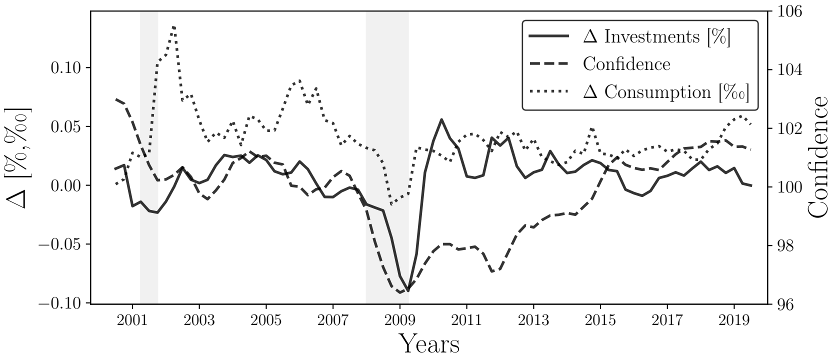

Figure 8: Trajectories of the OECD confidence index, changes in consumption [‰], and investment [%] in the United States over a period from the beginning of 2000 to the present. The data were taken from FRED and OECD.

The implementation of rational expectations mathematically translated into the Euler equation[13]. The Euler equation together with the Taylor rule re-defined the way interest rates are computed[14] as, joined together, they provide a solution for the interest rates that depend on the inflation's future expectations (see Sec.(2.2) for details). The last key element introduced by the interventionist community is the implementation of the real business cycle (RBC) theories, *via* the addition of an exogenous shock to the aggregate output of the economy[15]. If one assumes the output's amplitude is modulated by an external perturbation, called technology, following an Auto Regressive (AR) stochastic process, then the result reproduces the oscillating pattern of the business cycle. O.Blanchard [72] finds in this last category of models, and in particular the E.C. Prescott [73] Real Business Cycle model, the birth of the New Keynesian Dynamics Stochastic General Equilibrium (DSGE) models. Ever since, their formulation, DSGE models have been the workhorse for central banks and policy makers [74, 75]. During the years, DSGE models have developed and their more recent formulations follow Christiano [76] , F. Smets and R. Wouters [77] and others [78–80].

2.1.3 *The 2008 Global Financial Crisis*

In the same way the great depression of the 1930s challenged the invisible hand, or the great inflation of the 1970s challenged the Neoclassic approach, the global financial crisis of 2008 marked a profound

13 The result of the forward-looking utility function maximisation, see Sec.(2.2) for insights

14 Before the Lucas critique [46] expected inflation was computed starting from past inflation's rate.

15 With a modern outlook introducing exogenous shocks with $\approx 100\%$ auto-correlation to generate cycles that otherwise would not exist otherwise, is not very "evolutionist".

break in the economic community[16]. Economists realised that the DSGE models not only failed to predict the economic crisis but, as Stiglitz said:

"under their core hypothesis (rational expectation, exogenous shocks), a crisis of that form and magnitude simply couldn't occur."[85]

The gravity of the situation led to the denunciation of Jean-Claude Trichet, then governor of the European Central Bank:

"Models failed to predict the crisis and seemed incapable of explaining what was happening [...], in the face of the crisis we felt abandoned by conventional tools." [87]

In an attempt to appease some of the fierce criticisms that were pronounced after the global financial crisis against the Neoclassical models, the economic research community worked tirelessly adding key parts that had been missing until then, such as, for example, the absence of a financial sector [87–89].

Similarly to the diatribe between evolutionists and interventionists that dominated the debate during the Great Inflation, following the evident failure of DSGE models during the 2008 GFC [85, 87, 89, 90] the debate has been fierce [91] and new approaches to modelling the economy have started to arise.

To try to overcome the DSGE doctrine, a faction of even more "revolutionary" economists is pushing for the classical doctrine to be completely abandoned and replaced by agent-based models (ABM). Their efforts are devoted to overcome the axiomatic nature of macroeconomic models. The change in the paradigm seems, this time, to be radical. In contrast, the second group, more popular, still remaining sceptical about the efficiency of the current DSGE formulation, believe in their improvement, including for example the financial markets [92], and their central role in the future of Macroeconomics[72, 90, 93].

The following part of the chapter is devoted to the introduction of the DSGE framework. It will be used as a reference for the second part of this thesis, where I present the work I have conducted during the last years. Its structure is divided into three main sections. In Sec.2.2, as it is the starting point of my research, I present and solve the simplest monetary model. The following section, Sec.2.3 is devoted to two relevant extensions. First, I discuss how the capital is implemented in the benchmark DSGE. In particular, I show how the household's budget constraint and the production function of the representative firm are modified when considering the existence of a

16 See the extensive discussions in [72, 81–85], with a recent review in [86].

capital market. This setup is not completely solved, but all the key features are discussed. The second expansion is the integration of heterogeneities within the DSGE formulation. The analysis focuses on the so-called Two-Agent New Keynesian models (TANK) and the Heterogeneous-Agent New Keynesian (HANK) models, which are compared to the Representative-Agent New Keynesian (RANK) framework typical of the benchmark DSGE formulation. Since TANK models are a special case of the HANK formulation, the accent will be put on the latter. Again, the benchmark HANK model will not be entirely solved, but only the key ingredients are presented. These two extensions serve to form an intuition on how those issues are tackled by the DSGE community. Finally, the third section, Sec.2.5 treats the main differences between DSGE and ABM models. The discussion focuses on what have been recognised (by the community) as the main shortcomings of both schools of thought. On the one hand, the ABM framework seems to be the natural answer to some issues of the DSGE models, while, on the other hand, there still are some concerns about this newer approach. The discussion will focus on four main topics, which are inevitably interconnected and are:

- The versatility of DSGE/ABM to model endogenous and/or exogenous phenomena (such as economic crises)
- The general equilibrium principle on which DSGE models are based
- The representative agent framework, whose formulation cancels interactions and heterogeneities, versus multi agents modelling
- The axiom of rationality

During the discussion, and well aware of ideological fractures, I will try to be critical of both approaches, highlighting their limitations and strengths.

2.2 THE SIMPLEST MONETARY MODEL

A Dynamics Stochastic General Equilibrium model models the relationships between the major actors taking part in the business cycle (representative household, a representative firm and the central bank etc.) and sets some rules from which the most relevant aggregate macroeconomic quantities (as consumption, labour, wages etc.) are computed. More specifically, the goal of DSGE models is to determine how exogenous shocks propagate within the system and affect aggregate quantities.

A rough sketch of the interactions of this DSGE is shown below, see Fig.9.

The following setup will serve as a starting point for the next chapter, Ch.3, where we show how the addition of simple behavioural

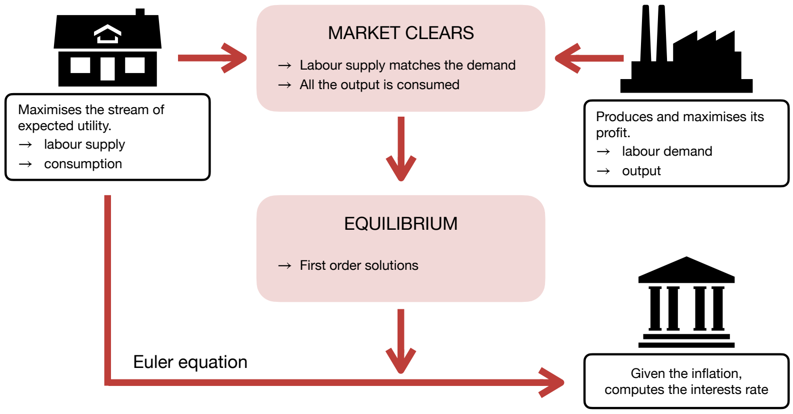

Figure 9: A schematic representation of the most simple DSGE model. The graph shows how the household, the firm and the central bank interact.

rules can dramatically change the phenomenology. Similarly to the mean field approach of the Ideal gas law equation, the DSGE models investigate the evolution of aggregate macroeconomical quantities, e.g. consumption or labour.

2.2.1 *The Representative Household*

Every period t, the infinitely living representative agent maximises its stream of expected utility U_t over an infinite, discrete, time horizon. The utility function encodes the preferences of the household regarding its aggregate consumption c_t and its propensity to supply labour n_t to the firm. Every time period t, the household's utility function reads:

$$U(c_t, n_t) := \frac{c_t^{1-\sigma}}{1-\sigma} - \gamma \frac{n_t^{1+\phi}}{1+\phi}, \tag{11}$$

where γ is a disutility term coupled to labour, σ and ϕ two exponents that modulate the concavity of the utility function respectively. The utility function has an increasing concave dependence on the consumption term, and it is decreasing and convex (and negative) on the labour N_t.

A frequent choice is to take the two limits $\sigma \to 1$ and $\phi \to 1$, which translates into a logarithm dependence of the utility function on the aggregate consumption and a quadratic relation on labour.

The household's maximisation of its utility is constrained by its budget. The budget constraint compares the incomes of the household to its outcomes, with the latter constrained to be smaller.

The income consists of the nominal wage w_t that the firm pays to the household for unity of labour supplied n_t. The second source of income comes from the bonds' market. Every period the representative agent receives b_{t-1} representing the amount of one period bonds bought at time $t-1$ at a price $(1+r_{t-1})^{-1}$, with r_t being the interest rate, and having matured at time t. Every time step, the household buys goods at a price p_t and purchases bonds. The budget equation therefore reads:

$$p_t c_t + \frac{b_t}{(1+r_t)} \leqslant w_t n_t + b_{t-1} + E_t \ , \tag{12}$$

where the term E_t represents other external sources of income (as for example dividends, taxes etc. etc.). In addition, to avoid any possibility for infinite borrowings (also known as Ponzi schemes), it is imposed the following constraint on bonds:

$$\lim_{t' \to \infty} \mathbb{E}_t[b_{t'}] > 0, \quad \forall t \ . \tag{13}$$

Finally, the infinitely rational household maximises the stream of its expected discounted (a term of patience $\beta = 1-\rho$, with $\rho \ll 1$ is introduced) utility under the budget constraint:

$$\begin{aligned} \max_{\{c_t, n_t, b_t, \lambda_t\}} \Lambda = \max_{\{c_t, n_t, b_t, \lambda_t\}} \mathbb{E}_t \Bigg[\sum_{t'=0}^{\infty} \beta^{t'} \mathcal{U}(c_{t'}, n_{t'}) \\ + \beta^{t'} \lambda_{t'} \left(p_{t'} c_{t'} + \frac{b_{t'}}{(1+r_{t'})} - w_{t'} n_{t'} - b_{t'-1} + E_{t'} \right) \Bigg] \end{aligned} \tag{14}$$

This maximisation is achieved exploiting the Lagrange multiplier's method and yields to a system of 4 equations shown below.

$$\partial_{c_t} \Lambda = 0 \quad \to \quad p_t \lambda_t + c_t^{-\sigma} = 0 \tag{15}$$

$$\partial_{n_t} \Lambda = 0 \quad \to \quad -\gamma n_t^{\phi} - \lambda_t w_t = 0 \tag{16}$$

$$\partial_{b_t} \Lambda = 0 \quad \to \quad \frac{\lambda_t}{(1+r_t)} - \beta \mathbb{E}_t \left[\lambda_{t+1} \right] = 0 \tag{17}$$

$$\partial_{\lambda_t} \Lambda = 0 \quad \to \quad p_t c_t + \frac{b_t}{(1+r_t)} - w_t n_t - b_{t-1} - E_t = 0 \ . \tag{18}$$

The equation, Eq.(15), sets the Lagrange multiplier to $\lambda_t = -c_t^{-\sigma}/p_t$. Injecting its value into Eq.(16) follows the derivation of the household state equation:

$$c_t^{\sigma} n_t^{\phi} = \frac{\omega_t}{\gamma} \ , \tag{19}$$

where $\omega_t$represents the real wage and $\omega_t := w_t/p_t$. The combination of the two equations Eq.(15) and Eq. (17) gives the Euler equation:

$$c_t^{-\sigma} = \beta(1+r_t)\mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{1+\pi_{t+1}} \right] \ , \tag{20}$$

where π_t is the inflation defined as $\pi_t := p_t/p_{t-1} - 1$.

It should be intuitive, for the reader, how rational expectations are encoded in the Euler equation. This intertemporal optimisation is derived by the infinitely rational household that forms its consumption decisions at time t by estimating both the future inflation and its proper future consumption.

2.2.2 *The Representative Firm*

The representative firm hires the household's labour and produces its output according to a Cobb-Douglas production function Y_t:

$$Y_t = z_t \frac{l_t^{1-\alpha}}{1-\alpha} \,, \tag{21}$$

where the exponent α modulates the relationship between output and labour. A common choice is to set $\alpha = 1/3$. Again, the model presented here is the simplest setting possible, as the capital is absent. Later in this document, I will introduce its extension to capital and other types of production functions. The production function has a concave dependence on the labour demanded to the household, say l_t, and it is linearly modulated by an exogenous technological shock z_t. As mentioned, the introduction of such exogenous shocks has its roots in the need of reproducing the patterns of the business cycle.

In order to better reproduce the RBC patterns, the exogenous shock is taken of the form $z_t := \bar{z}e^{\xi_t}$. The log-productivity ξ_t follows an AR(1) process:

$$\xi_t = \eta_z \xi_{t-1} + \sqrt{1-\eta_z^2}\,\mathcal{N}\left(0, \sigma_z^2\right) , \tag{22}$$

where η_z modulates the temporal correlations of the technology shocks and σ_z the amplitude of these shocks.

The firm seeks to maximise its real profit $\mathbb{P}_t/p_t$:

$$\mathbb{P}_t/p_t = Y_t - \omega_t l_t \,, \tag{23}$$

which compares a general level of demand Y_t and the firm's labour demand l_t times its real cost $\omega_t := w_t/p_t$.

The firm maximises Eq. (23) with respect to the labour demand l_t, or:

$$\partial_{l_t} \mathbb{P}_t/p_t = 0 \,. \tag{24}$$

This gives the real wages as a function of a general level of labour l_t :

$$\omega_t = z_t l_t^{-\alpha} \,. \tag{25}$$

The two missing ingredients necessary to close the model are (i) the assumption that markets clear and (ii) the general equilibrium hypothesis. These assumptions are extremely strong and not exempt from criticisms.

2.2.3 *Market clearing*

Imposing that both the labour and goods markets are in equilibrium allows closing the model. More precisely, both the labour and the goods marker clear simultaneously:

$$\begin{aligned} l_t &= n_t & (26)\\ c_t &= Y_t \,. & (27)\end{aligned}$$

This allows to match the labour demand with the offer, Eq.(26), and to equate the aggregate consumption of the household with the output of the firm, Eq.(27).

Combining Eqs. (25) and (26) together with Eq. (19) provides a self-consistent equation for the aggregate consumption:

$$n_t^{\phi+\alpha} c_t^{\sigma} = \frac{z_t}{\gamma}\,, \qquad n_t = \left[\frac{(1-\alpha)c_t}{z_t}\right]^{\frac{1}{1-\alpha}}\,. \tag{28}$$

Mathematically speaking the macro economical aggregate variables can already be computed, but it is necessary to assume the existence of one unique equilibrium to set the relations between macroeconomical variables and the exogenous shock ξ_t.

2.2.4 *General Equilibrium*

The general equilibrium paradigm describes the trajectories of the macroeconomical variables as a series of immediately achieved equilibria. This translates into a linearisation of the Eqs. (28) and (20). The analysis of this model is achieved comparing the first order expansions of the aggregate quantities:

$$\begin{aligned} c_t &= \bar{c}(1+\gamma_t)\\ n_t &= \bar{n}(1+\nu_t)\\ \omega_t &= \bar{\omega}(1+\delta_t)\\ z_t &= \bar{z}(1+\xi_t)\\ \beta &= 1-\rho\,, \end{aligned} \tag{29}$$

where the upper bar is used to distinguish equilibrium terms (time independent). Skipping some algebra, the injection of the expansions defined in Eq.(29) into Eq. (28) provide the equilibrium level of the macro economical quantities

$$\begin{aligned} \bar{c} &= \frac{1}{\gamma}\bar{z}^{A_{\bar{c},\bar{z}}}\,, \qquad A_{\bar{c},\bar{z}} = \frac{1-\phi}{(1-\alpha)\sigma+\phi+\alpha}\\ \bar{n} &= (1-\alpha)^{\frac{1}{1-\alpha}}\bar{z}^{A_{\bar{n},\bar{z}}}\,, \qquad A_{\bar{n},\bar{z}} = \frac{A_{\bar{c},\bar{z}}-1}{1-\alpha}\\ \bar{\omega} &= \bar{z}^{A_{\bar{\omega},\bar{z}}}\,, \qquad A_{\bar{\omega},\bar{z}} = 1-\alpha A_{\bar{n},\bar{z}}\,. \end{aligned} \tag{30}$$

The first order deviation from such equilibrium as a function of the exogenous AR(1) process ξ_t:

$$
\begin{aligned}
\gamma_t &= M_{\gamma,\xi} \cdot \xi_t \,, && M_{\gamma,\xi} = (2-\alpha)/(\phi+\alpha+\sigma-\alpha\sigma) \\
\nu_t &= M_{\nu,\xi} \cdot \xi_t \,, && M_{\nu,\xi} = \frac{A_{\gamma,\xi}-1}{1-\alpha} \\
\delta_t &= M_{\delta,\xi} \cdot \xi_t \,, && M_{\delta,\xi} = 1-\alpha M_{\nu,\xi} \,. \qquad (31)
\end{aligned}
$$

From Eqs.(31) it is easy to understand how the exogenous shocks propagates within the economy. To small shocks will correspond small fluctuations of the aggregate quantities in a *small shock small business cycle* fashion. Conversely, large shocks will linearly affect the macroeconomical variables. The remaining steps to close the model are the computing of inflation and the setting of the interest rates, but these should be addressed by the monetary policy

2.2.5 *The Central Bank*

The Central Bank fixes the interest rates taking into account the household's future expectations, encoded in the Euler equation Eq.(20). The first order expansion (the zero order is trivial) of Eq.(20) compares the future increment in the consumption $\Delta\gamma_{t+1} := \gamma_{t+1} - \gamma_t$, the in interest rates r_t and the expected inflation:

$$\sigma \mathbb{E}_t \left[\Delta\gamma_{t+1}\right] = r_t - \rho - \mathbb{E}_t \left[\pi_{t+1}\right] \,. \qquad (32)$$

As discussed in the previous section, the monetary policy exploits the Taylor rule to provide a relation between the inflation and interest rates. Here, the Taylor rule is simpler than in Eq.(10) and takes the form:

$$r_t = \rho + \Phi\pi_t \,, \qquad (33)$$

where Φ is a free parameter, related to a_π previously defined. The most general Taylor rule, as proposed in Eq.(10), computes the interest rates looking at the output gap of the economy, i.e. the difference between the output of the firm and the optimal output of the economy, and the distance between current inflation and target inflation. Within this model, the output gap vanishes and accounting for a target inflation is irrelevant as the economy is price-less. Therefore, accounting to the simplifications of the present model, the Taylor rule reads as in Eq.(33).

Merging Eq.(32) together with Eq.(33) yields:

$$\Phi\pi_t = \mathbb{E}_t \left[\pi_{t+1}\right] + \hat{r}_t \,, \qquad \hat{r}_t := \sigma \mathbb{E}_t \left[\Delta\gamma_{t+1}\right] \,, \qquad (34)$$

where $\mathbb{E}_t \left[\Delta\gamma_{t+1}\right] = M_{\gamma,\xi}\mathbb{E}_t[\xi_{t+1} - \xi_t]$ can be expressed in terms of the noise, exploiting the relations found above, Eqs.(31).

Being the distribution of ξ_t known, the expected log-technology variation can be computed as $\mathbb{E}_t[\xi_{t+1} - \xi_t] = -(1-\eta)\xi_t$. Finally, $\hat{r}_t$ is set by:

$$\hat{r}_t = -\sigma(1-\eta)M_{\gamma,\xi}\xi_t \; . \tag{35}$$

The equation Eq.(34) can be solved in the two following regimes:

CASE $\Phi < 1$: In the regime $\Phi < 1$ the inflation at period $t+1$ reads:

$$\pi_{t+1} = \Phi\pi_t - \hat{r}_t + \zeta_{t+1} \; , \tag{36}$$

where ζ_t is a sequence of exogenous shocks satisfying $\mathbb{E}[\zeta_t] = 0, \forall t$[17]. This solution states that inflation can't be controlled by the monetary policies, as its value is intrinsically affected by a random variable ζ_{t+1}. To overcome this issue Φ is taken to be greater than unity.

CASE $\Phi > 1$: when $\Phi > 1$ inflation is computed by solving forward in time Eq. (34). This result reminds of a path integral, where inflation is the result of all the possible trajectories of the noise, weighted by their probability of occurrence. In this limit, the inflation reads:

$$\pi_t = \sum_{k=0}^{\infty} \Phi^{-(k+1)}\mathbb{E}_t\left[\hat{r}_{t+k}\right] = -\sigma(1-\eta)M_{\gamma,\xi}\sum_{k=0}^{\infty} \Phi^{-(k+1)}\mathbb{E}_t\left[\xi_{t+k}\right] \; , \tag{37}$$

where it has been exploited the explicit form of $\hat{r}_t$ given in Eq.(35). Exploiting the fact that $\mathbb{E}\left[\xi_{t+1}\right] = \eta\xi_t$, the value of the inflation is found, as a function of ξ_t, by solving a simple geometrical series:

$$\pi_t = -(1-\eta)\frac{M_{\gamma,\xi}}{\Phi - \eta}\xi_t \; . \tag{38}$$

The choice of $\Phi > 1$ is also known as the *Taylor principle*. We have seen, in the previous section, how, following the introduction of the Taylor rule, Central Banks reacts to inflation. Here is a practical example: setting $\Phi = 1.5$ and $\rho = 0.1$, when inflation grows, say by 1%, the interest rates must grow by $r_t = 1.6\%$ so the real interest rates (computed as the difference between nominal interest rates and the inflation) are positive.

With Eq.(38) the simplest monetary model is concluded and all the aggregate quantities are computed as a function of the exogenous shocks. The figure Fig.10 shows the trajectory of the log-consumption ($\log c_t$) versus the technology shocks ξ_t (being $\bar{z} = 1$), over 200 periods. This trivial sketch of the trajectories visually shows how the consumption oscillates around the equilibrium and any deviation from

17 So that taking $\mathbb{E}[\star]$ on both sides of Eq.(36), equation Eq.(34) is recovered

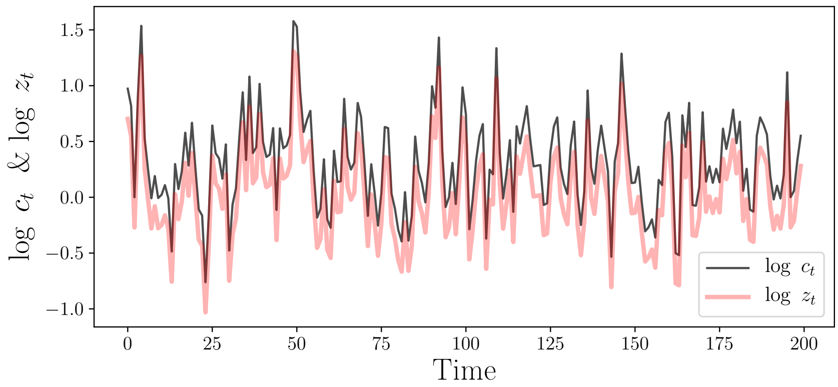

Figure 10: The figure sketches the dynamics of the log-consumption (solid black line) versus the log-technology (solid faded red line) for the simplest DSGE model, over 200 periods. Parameters chosen are: $\sigma = \phi = 1$, $\alpha = 1/3$, $\bar{z} = 1$ and $\eta_z = \sigma_z = 0.5$.

it is fully explained by the technological shocks. This simple DSGE framework allows for Business Cycles whose magnitude and duration is only determined by the parameters of the noise, i.e. σ_z and η_z. Within this benchmark monetary model, the economical recessions are not well-defined and even if they were, they are only driven by some particularly low realisations of the exogenous shocks. In the next sections, I present two extensions of this simple framework: (i) the addition of capital and (ii) the implementation of heterogeneities.

2.3 BEYOND THE SIMPLEST MONETARY MODEL

Over the years and especially after the 2008 GFC, DSGE models have considerably evolved. The main post-crisis objective has been the implementation of the financial market into the business cycle, see [94] and its frictions, see for example [95, 96]. Other related directions consider the real estate market [97, 98] and its implications. Of all the possible directions, I only consider two (important) ones: the implementation of the capital market (pre-existing to the GFC) and the introduction of heterogeneities.

These two directions follow the natural development of my research work. In the second part of this thesis and in particular in chapter Ch.3 I will present a modification of the simplest DSGE model which includes a simple feedback mechanism and displays some economical recessions due to confidence collapse. In the following chapters we will modify this model by adding first heterogeneity, Ch.4 and then the capital, Ch.5. The two extensions discussed below will allow a better comparison between our results and the more "standard" DSGE predictions. In order to provide a clearer explanation, the capital market extension will be presented before the heterogeneous one, in a reverse order compared to the second part of this thesis.

2.3.1 *The Addition of the Capital Market*

The first extension to the DSGE model presented in Sec.2.2 is the addition of the capital market. The aim of this section is to describe how the capital k_t enters the DSGE framework. The setup presented here is inspired by Jergen and Roehe [99, 100] and J.Gali [78, 80]. The original model accounts for a continuum of intermediate firms $i \in [0, 1]$ producing a distinct good i, purchased by the household at a price $p_t(i)$. In the following this sophistication is neglected as, in the next chapters, we will not account for this diversity. The implementation of the capital market is therefore focused on the stylised version of this setup, where the firm is representative. As mentioned at the beginning of the chapter, the goal of this part is not to solve the model, but to provide the reader with an understanding of how the capital market is implemented into the framework.

2.3.1.1 *The Household*

Similarly to the previous setup, the household maximises the same utility function described in Eq.(11), with the addition of an investments/savings (IS) exogenous shock[18] modulating the propensity of the household to consume. For the sake of clarity we will neglect this terms, but it is useful to mention its existence. The budget constraint, on the other hand, undergoes some major changes. Similarly to Eq.(12), the household purchases b_t bonds at a price $(1 + r_t)^{-1}$, holds b_{t-1} bonds that have matured, consumes c_t and provides n_t hours of work to the firm that pays w_t. The household also has some external source of income, labelled E_t. The new budget equation reads:

$$p_t c_t + p_t i_t + \frac{b_t}{(1 + r_t)} + p_t \phi_k \left(\frac{k_{t+1}}{k_t} - 1 \right)^2 k_t \leqslant \\ w_t n_t + b_{t-1} + q_t k_t + d_t \ , \quad (39)$$

where x_t is an exogenous term representing the efficiency of the investment. The distribution of x_t is given by:

$$\log x_t = \rho_x \log x_{t-1} + \mathcal{N}(0, \sigma_x^2) \ , \quad (40)$$

with $0 < \rho_x < 1$ being a parameter that regulates its correlation. The capital, k_t depreciates at a constant rate δ and follows a dynamic given by:

$$k_{t+1} = (1 - \delta) k_t + x_t i_t \ . \quad (41)$$

18 Always considered to be of the form $\log a_t = \rho_a \log a_{t-1} + \mathcal{N}(0, \sigma_a^2)$.

The novelties of Eq.(39) are: (i) the household supplies k_t units of capital to the firm that pays revenues q_t on it and (ii) to compensate for the capital depreciation, Eq.(41), the household invests an amount i_t in the capital market paying a penalty given by its adjustment cost:

$$\phi_k \left(\frac{k_{t+1}}{k_t} - 1 \right)^2 k_t \,, \tag{42}$$

where $\Phi_k \geqslant 0$ represents the amplitude of the friction. Third, at every period, the firm pays to the household its dividends d_t, as the latter is the owner.

2.3.1.2 *The firm*

The firm's output level Y_t is modulated by the capital level k_t and the labour provided by the household n_t. The standard approach considers a Cobb-Douglas production function of the form:

$$Y_t = z_t \frac{n_t^{1-\alpha} k_t^{\alpha'}}{1-\alpha} \,, \tag{43}$$

where α and α' are exponent, determining the nature of returns to scale. To ensure constant returns to scale[19] the sum $1 - \alpha + \alpha'$ is fixed to unity. The overall productivity z_t is defined as in Eq.(21) and exogenously determined. Thus, the production function reads:

$$Y_t = z_t \frac{n_t^{1-\alpha} k_t^{\alpha}}{1-\alpha} \,. \tag{44}$$

As a parenthesis: the Cobb-Douglas production function has some downsides: if the capital level drops, the firm, to ensure a constant output, compensates the lack of capital by increasing the labour demand (labour demand and labour supply match as markets clear). To completely avoid this mechanisms one can introduce the Leontief production function, i.e. $Y_t = z_t \min(n_t, k_t)$ and the output is limited when the capital is scarce. Through the Cobb Douglas production function the firm, in order to keep the output level constant, can make up for the lack of capital by investing in the labour market. Hiring more labour will eventually compensate for the scarcity of capital. The classic example that could be used as a counter-argument is that when one wants to build 1 car from 3 wheels, they simply can't. This obvious result does not change even if the firm doubles the working hours.

In the last chapter of this thesis, Ch.5, we investigate new possibilities for the production function. In particular, we will explore the Constant Elasticity of Substitution (CES) production function, of which Cobb-Douglas and Leontief are particular limits.

19 Whenever the labour supplied by the household and the capital level simultaneously double, the output doubles as well.

As in the benchmark model, the market clears and the household consumption equates the firm's output. The firm seeks to maximise its real profit, $\mathbb{P}_t/p_t$, with respect to the labour n_t the capital level k_t and the price p_t.

Any price variation undergoes some frictions, modulated, similarly to the adjustment for capital, by a quadratic term proportional to:

$$\frac{1}{2}\Phi_p\left[\frac{p_t}{(1+\pi)p_{t-1}}-1\right]^2 Y_t\ , \tag{45}$$

where π is the target inflation, and $\Phi_p \geqslant 0$ an overall amplitude term. It is worth mentioning that when $\Phi_p = 0$ price stickiness decays and economy becomes price-less as all the quantities can be expressed in real terms. The real profit, $\mathbb{P}_t/p_t$ reads:

$$\mathbb{P}_t/p_t = Y_t - \omega_t n_t - q_t k_t - \frac{1}{2}\Phi_p\left[\frac{p_t}{(1+\pi)p_{t-1}}-1\right]^2 Y_t\ . \tag{46}$$

As the household owns the firm, the term d_t appearing in the budget, Eq.(39), coincides with the nominal profit, i.e. $d_t = \mathbb{P}_t$.

2.3.1.3 *The Monetary Policy*

Finally, the monetary policy is represented by a Taylor rule that can be written as in Eq.(10), setting $a_\pi = a_y = \frac{1}{2}$, with an additional exogenous shock e_t:

$$r_t = r + \pi_t + \frac{1}{2}(\pi_t - \pi) + \frac{1}{2}\left(\frac{Y_t - Y}{Y}\right) + \log e_t\ . \tag{47}$$

We recall Y is the optimal output level, π the target inflation and r the ideal interest rates.

It is mandatory to add some remarks. First, the model here presented has some differences from most complex (and complete) versions (see below). It features perfect competition and flexible prices in all the markets. As a matter of fact, the whole economy can be re-scaled by the price and expressed in terms of real quantities. The more sophisticated versions of DSGE account for nominal rigidities. For example, a simple way of implementing them would be achieved by allowing the firm to fix the prices of the goods (multiple), but any price change would cost a penalty in the profits (generally taken quadratic, see [78, 80] for references). Secondly, this setup only considers the exogenous technology shocks that affect the global output of the firm. Other shocks might be added to the monetary policies and to the household's preferences, via financial markets, for example. Those extensions go beyond the scope of this dissertation and are not discussed here.

The versions of the DSGE models that are nowadays exploited by the institutions –although not (yet) including heterogeneities, an argument treated in the next section– include many features that are not presented here. At the beginning of Sec. 2.5, we will discuss how the most updated DSGE model performs when calibrated with real data. Its results will be compared to both real data (Austria) and to the performance of an equally complex Agent-Based Model.

2.3.2 *The Integration of Heterogeneities. TANK and HANK Models*

Although reality displays a vast number of examples where heterogeneities play a crucial role in the economic analysis, for instance the power-law distributed incomes and firms' sizes, within the Representative Agent New Keynesian paradigm (RANK) of the benchmark DSGE heterogeneities are clearly missing.

During the years, many attempts have been made to include different categories of households, for instance in the Two-Agent New Keynesian models (TANK) [101–104], or heterogeneous households with a continuum of possible accumulated wealth, as in HANK (Heterogeneous Agent New Keynesian) models [105–110] see also [111] for a different approach leading to emergent heterogeneities.

The TANK framework, see [104, 112] for insights, models an economy composed by two agents: the *hand-to-mouth*[20] household does not have access to the bonds market and consume the entirety of their income, is opposed to the *well-off*[21] household which, instead, invests in the bonds market. The same contrast also appears in the HANK models, which, however, are not based on two discrete participants, but rather on a continuum of constrained and unconstrained households. The TANK framework is therefore a discrete limit of the HANK, which is definitely more general. Because of its greater versatility, I introduce the HANK setup, leaving for the conclusions the comparison with the TANK framework.

The sketched Heterogeneous-Agent New Keynesian model discussed in this section is inspired by the model presented by D. Debortoli and J.Gali [104] in which they condense the relevant literature, see also [113–115] and [116–118]. In the following, I put the accent on those factors encoding heterogeneities in the HANK framework, trying to grasp the salient points of their formulation. Let's consider an economy with a continuum of households $s \in [0, 1]$. Each household is represented by a utility function U of the form of Eq.(11), where the individual labour's disutilities $\gamma_t^i \equiv \gamma_t$ are identical among agents and exogenous. Similarly to the previous setups, households are in-

20 Also referred in literature as *non - Ricardian*
21 Also referred as *Ricardian*

finitely rational entities maximising the stream of their expected utility over a discrete and infinite time horizon, or:

$$\mathbb{E}_t \sum_{t'=0}^{\infty} \beta^{t'} U(c_{t'}(s), n_{t'}(s)|\gamma_{t'}) \ , \quad \forall s \ , \tag{48}$$

where $\beta < 1$ is a patience term. The consumption of household s at time t is noted as $c_t(s)$ and the amount of working hours it supplies to the firm $n_t(s)$. As a short digression it is important to remark that within this framework, households are *ex ante* all equals as they all have the same preferences (utility functions). Heterogeneities are introduced by assuming that every period t, a portion λ_t (this choice has its root in [119–121]) of household are hands-to-mouth as they have constrained access to the bonds market. The remaining $1 - \lambda_t$ portion of the population has instead unconstrained access to the bonds market. This latter group is subject to the standard Euler, Eq.(20)[22]. The unconstrained and constrained sets at time t are noted $\mathbb{U}_t$ and, $\mathbb{C}_t$ respectively. For the sake of clarity, every household $s \in \mathbb{U}_t$ buys one period risk-less bonds b_t at a price $(1 - r_t)^{-1}$, with r_t being the interest rates. *Vice versa*, if the household is constrained, $s \in \mathbb{C}_t$ it can't. Thus, for any $s \in \mathbb{U}_t$, the Euler equation reads:

$$c_t^{-\sigma}(s) = (1 + r_t)\beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}(s)}{1 + \pi_{t+1}} \right] \qquad \forall s \in \mathbb{U}_t \ . \tag{49}$$

Although the derivation of the individual Euler equation Eq.(49) follows straightforwardly from Eq.(20), the quantities of interest are aggregated. As the constrained group does not have access to the bonds market, it is impossible to derive an Euler equation for this second set. Thus, writing the Euler equation for the aggregate consumption (unconstrained + constrained) is:

- Writing the aggregate Euler equation for the unconstrained set of households.
- Re-expressing the aggregate consumption in terms of the consumption of the constrained households.
- Deriving the aggregate Euler equation.

STEP 1: AGGREGATE UNCONSTRAINED EULER EQUATION. The average aggregate consumption of the unconstrained households $c_t^{\mathbb{U}}$ is defined as follows:

$$c_t^{\mathbb{U}_t} = \frac{1}{1 - \lambda_t} \int_{s' \in \mathbb{U}_t} c_t(s') ds' \ , \tag{50}$$

22 We recall it is derived from the maximisation of the stream of expected utility with respect to the bonds, see Eq.(17).

where $1-\lambda_t$ is the proportion of unconstrained households out of the full spectrum and Eq.(49) holds for each unconstrained households. Passing from the individual to the aggregate formulation requires the definition of $c_t^{\mathbb{U}}$. Integrating both the RHS and the LHS sides of Eq.(49) with respect to $s \in \mathbb{U}$, and after some mathematical manipulations[23], one can isolate the terms representing the aggregate consumption of the unconstrained group for the two successive time periods: $c_t^{\mathbb{U}}$ and $c_{t+1}^{\mathbb{U}}$.

The Euler equation for the unconstrained group reads:

$$\left(c_t^{\mathbb{U}}\right)^{-\sigma} = (1+r_t)\beta\mathbb{E}_t\left[\frac{\left(c_{t+1}^{\mathbb{U}}\right)^{-\sigma}}{1+\pi_t}\Theta_{t+1}\right] \tag{51}$$

$$\Theta_{t+1} = \frac{\int_{s'\in\mathbb{U}_t}\left(c_{t+1}(s')/c_{t+1}^{\mathbb{U}}\right)^{-\sigma}ds'}{\int_{s'\in\mathbb{U}_t}\left(c_t(s')/c_t^{\mathbb{U}}\right)^{-\sigma}ds'}\,, \tag{52}$$

where the multiplicative factor Θ_{t+1} appearing in Eq.(51) collects the integral terms and encodes the consumption heterogeneities within the set of unconstrained households.

To understand the meaning of the term Θ_{t+1}, D.Debortoli and J.Gali [104] provide a second order expansion of Θ_{t+1}, noted $\Theta_{t+1}^{(2)}$, which reads:

$$\Theta_{t+1}^{(2)} \propto \frac{\mathbb{V}_t[\log c_{t+1}(s)]}{\mathbb{V}_t[\log c_t(s)]}\,, \tag{53}$$

where the variance $\mathbb{V}_t[\cdot]$ represents the volatility.

The formulation of $\Theta_{t+1}^{(2)}$ provides an intuition over the economical meaning of Θ_{t+1}. In fact, when $\Theta_{t+1} = 1$ the Euler equation for the unconstrained group, Eq.(51), would read the same as the Euler equation, Eq.(20), derived in RANK scenario, see Sec.2.2. Grasping the meaning of Θ_{t+1} therefore translates to understanding the role of heterogeneities within the unconstrained group.

From Eq.(53) one realises that, *ceteris paribus*, Θ_{t+1} encodes the uncertainties of the unconstrained group to the future consumption. In fact:

- When uncertainties over the future consumption increase $\mathbb{V}_t[\log c_{t+1}(s)] > \mathbb{V}_t[\log c_t(s)]$, then $\Theta_{t+1} > 1$. From Eq.(51), fixing $c_{t+1}^{\mathbb{U}}$ to a constant, the average aggregate consumption $c_t^{\mathbb{U}}$ decreases as a form of precautionary savings.

- *Vice versa*, when future estimations are "safer", $\theta_{t+1} < 1$ and the average aggregate consumption $c_t^{\mathbb{U}}$ increases.

STEP 2: AGGREGATE EULER EQUATION. The total aggregate consumption c_t can be written as the sum of the contribution of the constrained and unconstrained households as:

$$c_t = (1-\lambda_t)c_t^{\mathbb{U}} + \lambda_t c_t^{\mathbb{C}}\,, \tag{54}$$

23 One has to multiply the LHS by $c_t^{\mathbb{U}}/c_t^{\mathbb{U}}$ and the RHS by $c_{t+1}^{\mathbb{U}}/c_{t+1}^{\mathbb{U}}$

where λ_t is the proportion of constrained households at the time t. The next ingredient is the definition of the average consumption gap:

$$\mu_t := (c_t^{\mathbb{U}} - c_t^{\mathbb{C}})/c_t^{\mathbb{U}} \in [0, 1] \tag{55}$$

[24] which encodes the consumption asymmetries between the two groups.

Being the equilibrium unique, it is possible to expand around it. Combining the definitions of the aggregate consumption c_t, Eq.(54) with the consumption gap Eq.(55), it is possible to express c_t as a function of $c_t^{\mathbb{U}}$, λ_t and μ_t, as:

$$c_t = c_t^{\mathbb{U}}(1 - \lambda_t \mu_t) \; . \tag{56}$$

Coherently to the notations introduced in Eq.(29), the first order corrections around such equilibrium are given by γ_t and $\gamma^{\mathbb{U}_t}$:

$$c_t = \bar{c}(1 + \gamma_t) \; , \qquad c_t^{\mathbb{U}} = \bar{c}^{\mathbb{U}}(1 + \gamma^{\mathbb{U}_t}) \; . \tag{57}$$

The ratio of constrained households and the consumption gap are also linearised around their value at equilibrium, say $\bar{\lambda}$ and $\bar{\mu}$ respectively. The deviations for such equilibrium are encoded into $\hat{\lambda}_t$ and $\hat{\mu}_t$, thus:

$$\lambda_t = \bar{\lambda} + \hat{\lambda}_t \; , \qquad \mu_t = \bar{\mu} + \hat{\mu}_t \; . \tag{58}$$

Expanding to the first order, Eq.(57) one finally obtains:

$$\gamma_t = \gamma_t^{\mathbb{U}} - \hat{\mu}_t \frac{\bar{\lambda}}{1 + \bar{\lambda}\bar{\mu}} - \hat{\lambda}_t \frac{\bar{\mu}}{1 + \bar{\lambda}\bar{\mu}} \; . \tag{59}$$

STEP 3: THE AGGREGATE EULER EQUATION. The aggregate Euler equation is obtained injecting Eq.(56) into Eq.(51). The HANK's Euler equation reads:

$$c_t^{-\sigma} = (1 + r_t)\beta \mathbb{E}_t \left[\frac{(c_{t+1})^{-\sigma}}{1 + \pi_{t+1}} H_t \right] \tag{60}$$

$$H_t = \left(\frac{1 - \lambda_{t+1}\mu_{t+1}}{1 - \lambda_t \mu_t} \right)^{\sigma} \Theta_{t+1} \; . \tag{61}$$

When one compares the two Euler equations, Eq.(60) for the HANK formulation and Eq.(32) of the RANK model, the only difference is the presence of an additional term H_t, which encodes the heterogeneities of the model.

The RANK limit is recovered when one only considers the unconstrained group of agents. This sets (i) $\lambda_t = 0$ as 100% of agents are unconstrained, (ii) $\mu_t = 1$ as $c_t^{\mathbb{U}} = 0$ and (iii) $\Theta_{t+1} = 1, \quad \forall t$. Thus,

24 The consumption of the constrained group is smaller than the unconstrained one, as the fact they don't invest in the bonds market is due to a limited income level.

in the RANK limit one has $H_t = 1, \forall t$ and the heterogeneities are switched off coherently with the representative agent DSGE model setup.

When one expands to first order the Euler equation Eq.(60), it is possible to isolate each of the contributions of the exogenous terms. Without lost of generality we set $H_t \sim 1 + h_t$, with h_t encoding the deviation from the RANK formulation:

$$h_t = \underbrace{-\sigma\frac{\bar{\lambda}}{1-\bar{\lambda}\bar{\mu}}\mathbb{E}_t\left[\mu_{t+1} - \mu_t\right]}_{h_t^\mu} \underbrace{-\sigma\frac{\bar{\mu}}{1-\bar{\lambda}\bar{\mu}}\mathbb{E}_t\left[\lambda_{t+1} - \lambda_t\right]}_{h_t^\lambda} + h_t^\theta \,, \quad (62)$$

where h_t^Θ represents the first order expansion of Θ_{t+1}.

In this way the individual contributions of h_t^μ, h_t^λ and h_t^Θ are separated and can be analysed separately. Their role can be summarised as:

- h_t^μ refers to the average consumption gap between unconstrained and constrained households;
- h_t^Θ measures the dispersion within the unconstrained group;
- h_t^λ accounts for the proportions of each group.

The TANK frameworks have two distinct representative agents, say one hands-to-mouth and the other one unconstrained, that forms the whole economy. This setup fixes (1/2, 1/2) the proportions between agents and lets the corresponding heterogeneity term vanish, i.e. $h_t^\lambda = 0$. Second, as the modellisation relies on the representative agent formulation, it doesn't allow for heterogeneities within the unconstrained group, negating the contributions to h_t of the corresponding term $h_t^\Theta = 0$. In other terms, the flux of agents from one group to the other is zero. The only upgrade TANK models can provide compared to RANK is the measure of the consumption gap between the two classes of agents. This allows for $h_t^\mu \neq 0$.

To illustrate to the reader the main results achieved by HANK models and to justify why their development has brought innovation within the New Keynesian models, I briefly present the results achieved by Kaplan et al. [106]. In their paper, they have developed a very sophisticated HANK model, including the financial sector. Thus, agents belonging to the unconstrained group have liquidity preferences through which they choose to allocate their investments between liquid and illiquid assets. Once calibrated to the US economy, this HANK model managed to reproduce the Gini coefficient of the country (for liquid/illiquid wealth) quite accurately, as reported in Tab.1. This result is an effect of heterogeneities in the returns on wealth, generated by the liquid/illiquid assets structure. In fact, a fraction of households ends up to frequently hold high-return illiquid assets, thus generating high-profits. When these states persist, the associated households move to the upper part of the wealth distribution[106].

	Liquid wealth		Illiquid wealth	
Moment	Data	Model	Data	Model
Top 0.1 percentage share	17	2.3	12	7
Top 1 percentage share	47	18	33	40
Top 1 percentage share	86	75	70	88
Bottom 50 percentage share	-4	-3	3	0.1
Bottom 25 percentage share	-5	-3	0	0
Gini coefficient	0.98	0.86	0.81	0.82

Table 1: The table compares the outcomes of the HANK model developed by G.Kaplan et al. [106] with real data. Table taken from the online version of their paper.

This is an important leap from the representative agent models as they cannot, by construction, take into account wealth distributions in any way. In the previous chapter, we saw how heterogeneities play a fundamental role in Physics. Similarly, they must be taken into account in the economic analysis, as their role may not be negligible. During Ch.4 we will address the problem of heterogeneities, suggesting an alternative framework considers them.

2.4 MAIN CRITICISMS ADDRESSED TO DSGE MODELS

To understand what key elements one has to add to the DSGE framework in order to bridge the gap with the ABM methodology, it is imperative to discuss the main conflict points. Among them, we decided to restrain the analysis to four open questions, which arose after the 2008 GFC, DSGEs have been asked to answer. In particular, we will focus on the general equilibrium assumption, trying to grasp what are the reasons behind its formulation, and what its consequences are. Another important point is the absence of endogenously driven phenomena; in fact, as discussed in Sec.2.2, any deviation from the unique equilibrium is fully explained by an exogenous term implemented in the model. The lack of multiple equilibria strongly clashes with the emerging aggregate behaviours naturally arising in nature, as discussed in Ch.1. Coordination between the individual elements of a flock of starlings, as well as the coordination of investors in the stock market (and many other examples), can only be explained when one considers endogenous processes generated by heterogeneities and interactions. The need to account for heterogeneities and interactions undermines the concept of representative agent rooted in the classical DSGE. Last, I discuss the axiom of full rationality. Although rational expectations are an efficient mathematical tool, they hide several problems. These four points are deeply interlinked and can hardly be solved one at a time.

2.4.1 *General Equilibrium, its Stability and Uniqueness*

DSGE models are by definition based on the assumption of a general equilibrium, i.e. the simultaneous equilibrium of all markets. In the DSGE framework, the trajectory of macroeconomic variables is formally modelled as a succession of immediately reached equilibria. To be more precise, any deviation from such equilibrium, see Sec.2.4.3, is purely exogenous and uncorrelated to the fundamental macroeconomic variables. More specifically, the DSGE paradigm is based on the fact that equilibria are unique and stable. Both the uniqueness and the stability of the DSGE equilibrium have a great impact on the analyticity of DSGE solutions, as their uniqueness allows one to linearise the macroeconomic variables around its value, and express them in terms of the external noise, see Sec.2.2. If the concept of unicity was undermined, for instance by the coexistence of several equilibria, the process of linearising the economic aggregate variables would lose its meaning. As a matter of fact, any sufficiently large shock would allow switching from one equilibrium to another and, therefore, its properties. In DSGE models, not only does a unique equilibrium exist, but also, it is stable. Stability allows shocks to have no macroscopic effect on solutions, and to be rapidly reabsorbed by the economy. Macroeconomical variables can, therefore, be expressed in terms of the exogenous shocks as they will maintain, over time, a constant magnitude. Following A.Kirman [44, 122], the uniqueness and the stability of the equilibria are intimately bonded to the representative agent paradigm, which will be discussed below in Sec.2.4.2. The complete absence of non-linearities from the DSGE framework makes it difficult to accommodate economical recessions which have their foundations in unstable self-reflexive mechanisms, as it is the case for the 2008 Global Financial crisis, for example. In fact, as a consequence of linearities, in a DSGE framework economical crises are only possible as a consequence of a large and persistent shock, in a *large shock large business cycle* fashion. Quoting O. Blanchard in [89]:

> *"We in the field did think of the economy as roughly linear, constantly subject to different shocks, constantly fluctuating, but naturally returning to equilibrium over time. [...]. The problem is that we came to believe that this was indeed the way the world worked."*

In order to provide a theoretical explanation for the formation of economic crises, it is imperative to go beyond the concept of general equilibrium, implementing those ingredients that in Physics as well as in Economics are main drivers of phase transitions: heterogeneities and interactions.

2.4.2 *The Lonely Representative Agent*

The effects of heterogeneities and interactions are completely missing in the classic DSGE models, as they only look at one representative actor (RA) maximising a utility function that condenses the global preferences [25]. Although it has some clear analytical benefits, the RA paradigm has several conceptual flaws, neglecting important effects that might be the underlying drivers of the economy.

In the same way as was advocated by Anderson in "*More is different*", the jump from individual to aggregate behaviour is not at all straightforward. With the same spirit, A. Kirman discusses the problem of the logical transition between the RA and the community. Quoting his words in "*Who or whom is the representative agent actually representing*"[122]:

"*Consider the most favourable situation, that in which we can construct an individual whose utility-maximising choices correspond to the aggregate choices of the individual in an economy. Even in this case, the representative agent can lead to misleading policy analyses. In models with a representative consumer, one makes the policy change and then examines the new equilibrium for the representative. However, there is an implicit assumption here that, after the change, the choice of the representative will still coincide with the aggregate choice of the individuals in the economy.*".

In other words, even in the optimal scenario where the representative agent is actually representing the population, any policy change might undermine his representative role. Second, in the introductory chapter it was mentioned, giving numerous examples, how non-trivial behaviours (phase transitions are the most striking manifestation thereof) and non linearities can emerge when aggregating many interacting agents. Those effects are completely missed by the original DSGE framework, as they are "averaged out" by the RA representation. For example, within the RA paradigm, it is impossible to explain how income inequalities affect the gross domestic product of a country. Without considering heterogeneities as an input, however, it is a hard task to explain this observation.

In the previous section, two frameworks that conserve the axioms of standard DSGE, but deviate from the RANK, have been presented: TANK and HANK models. On the one hand, it was discussed how TANK models do not really encode heterogeneities, as they only con-

25 This helps the DSGE framework to provide a stable and unique equilibrium [44, 122].

sist of two superposed RANK models. On the other hand, HANK models seem to provide a more intriguing response to the lack of heterogeneities. Within the HANK framework, households are *ex ante* all equal but *ex post* all different. In fact, preferences (utility functions) are the same within the whole spectrum of the population, and heterogeneities emerge as a response to the aggregate exogenous shocks and/or to different investments strategies. Although HANK models are definitely an interesting upgrade from the RA paradigm as they provide an explanation on how exogenous shocks affect inequalities, see Refs.[106, 123], one criticism that is often addressed to HANK is that they cannot, by construction, answer the reverse question: what is the impact of inequalities on the economy. In Ch.4, we provide our answer to this question building a DSGE-like model, where agents are associated with different skills levels that proportionally affect their wages.

2.4.3 *Exogenous and Endogenous Shocks. Black Swans and Dragon Kings*

The aim of DSGE models, and in particular all the real business cycle models[124–126], is to understand how exogenous shocks propagate through the economy, affecting macroeconomic variables such as consumption, labour, wages, etc. Considering the DSGE dynamics described by a series of immediately achieved equilibria, this framework does not allow for exogenous shocks to endure and amplify within the system. In fact, exogenous shocks are instantly reabsorbed in a *small shocks small business cycle* fashion. According to this perspective, severe economical crises are the direct consequence of rare and large technological fluctuations (*large shocks, large business cycle*). Those extreme exogenous events, also referred by N.Taleb as *black swans*[127], are so rare that probability of their occurrence is unknown, i.e. they cannot be predicted, but they have a catastrophic impact in the economy when they do occur. Within the DSGE paradigm, economical crises are fundamentally black swans. Referring to the 2008 GFC, R.Lucas [128] stated: "The 2008 crisis was not predicted because economic theory predicts that such events cannot be predicted."

When one tries to dig into the exogenous shock that triggered the GFC, they realise that the original subprime shock (estimated in 250 billions USD $) cannot explain alone the amplitude of the decline in the world stock market capitalisation (100 times more severe $\sim$ 26400 billions USD $), in what looks more like a *small shock large business cycle* scenario, as reported in Fig.11. In other words, the original subprime shocks endogenously amplified by two orders of magnitude. Such amplification is not negligible, and it is definitely impossible to explain it within a DSGE framework.

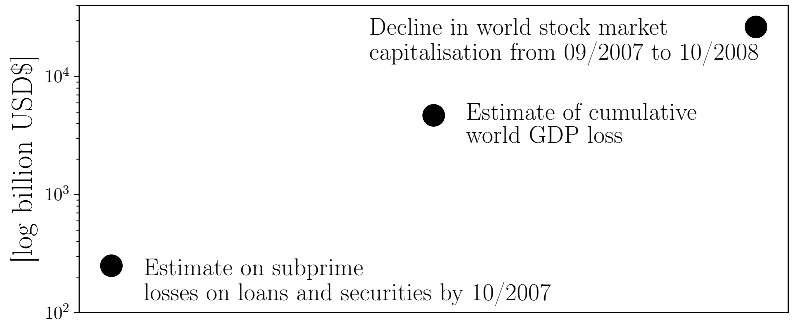

Figure 11: The graph compares the initial subprime losses (~ 250 billions USD$), the decline in the world GDP (~ 4700 billions USD$) and the drop of the world stock market capitalisation (~ 26400 billions USD$) between 2007 and 2008. The graph has been reproduced from data found on O.Blanchard's book: *The crisis: basic mechanisms and appropriate policies* [129]

The endogenous response of social systems and of financial markets to exogenous shocks have been of major interest in the last decades, see [130–132] and [133–135] for financial markets applications.

Diametrically opposed to the concept of black swan are the endogenously born *dragon kings*. A dragon king is an event whose magnitude cannot be explained by a probability distribution[26] and is generated by different mechanisms from their peers[137, 138]. Dragon kings are all those events endogenously amplified in a cliff-like behaviour that occurs, in non-linear systems, via feedback loops as, for example, confidence collapses, and are, by construction, completely missing within the DSGE paradigm. According to Sornette[137, 138], economical crises such as the 2008 GFC, are in fact dragon kings, and they can, to a certain extent, be predicted if the underlying feedback mechanisms are understood and implemented in the models. In Ch.3, we propose a modification to the most simple DSGE model, where we introduce a confidence feedback that endogenously amplifies the exogenous shocks, already well implemented in DSGEs, up to the point where panic effects trigger long and persistent economical collapses of the economic moments.

2.4.4 *Rationality and Future Forecasting. Animal Spirits & Unknown Knowns*

The DSGE representative agent maximises the stream of expected utility as a fully rational entity, as described in Eq.(14). In the same fashion as the chess player forecasts the expected moves of his opponent before moving, the RA interacts with many copies of himself and computes all the possible trajectories of the economy, setting his pref-

26 For example, the city sizes in France follow a Zipf's law, see Ref. [136], except for Paris, which is off scale (dragon king).

Figure 12: An example of rational behaviour at the stock market.

erences accordingly. Unfortunately for the chess player, the number of possibilities grows exponentially fast and having full control of the board is quite a hard task. Even world legendary players like Bobby Fischer are not completely rational, as Michail Tal reports in a nice anecdote:

"When I asked Fischer why he had not played a certain move in our game, he replied: 'Well, you laughed when I wrote it down.'".

One might argue that if your opponent laughs, it is rational not to play that particular move. However, in this example, the concept of absolute rational agent (as is the RA) is questioned. The *homo economicus* described by the DSGE models, instead, fully knows the mechanisms driving the economy and uses this knowledge to predict the future (although with a light patience term $\beta \sim 1$, see Eq.(14)). His capacity to anticipate the economy's outputs is prone to criticism: expectations can indeed be subject to rapid change, disagreement and irrationality, as reflected in the high volatility of investment [139], and the abrupt nature of expansions and recessions.

The axiom of rationality does not account for more complex (and sometimes irrational) behaviours of human nature, often referred in the literature as *animal spirits*[27] or *irrational exuberance*, see [141, 142] and [143–146]. Animal spirits as a driver of the decision process of the homo economicus were already observed by Keynes as, in the *General*

27 See [140] for an early review of animal spirits in macroeconomic models.

theory, [49] he reported:

> *"Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities."*

There is now rich literature on irrational behaviour across Economics (see Ref. [147] for a recent review). Despite Keynes's efforts, animal spirits as a driver of investors behaviour have not yet been included into more traditional business cycle models,[28] in what has been defined by Minsky [47] as an "*aborted revolution*". Some attempt of adding bounded rational components in DSGE models have been made during the years, such as Refs. [149–152] that focus on learning in expectations formation in a single-actor model, as well as Refs. [153, 154] that use various utility specifications in DSGE. But apart from some examples [155–157], there is surprisingly little work attempting to factor confidence or sentiment into the DSGE framework as an explicit variable.[29]

Another important ingredient that is missing from the DSGE framework are the *unknown-knowns*: those variables that not only we ignore, but over which we ignore our ignorance. Following the logic of Storm [161], rational maximisation over the future states of the economy is only possible, by definition, in the case where their probabilistic distributions are fully accessible to the agent. This implicitly implies that distributions exist, are well-defined, and are fully known and accessible. This brings up a very simple paradox: unfortunately, for the representative agent to omit the unknown-knowns from his forecasting process is very irrational. The orthodox school would answer this enigma, stating that agents behave *as if* they computed probabilities of things they don't know. Full rationality is one of the most criticised axioms of DSGEs, but, even in their most recent versions[77, 78, 162], they seem incapable to overcome this issue.

The inability, sometimes only partial, of the DSGE models to promptly answer these questions has led to a schism in macroeconomic thinking. In parallel with the improvements that have recently been made to DSGEs, a growing community of economists is proposing a rad-

28 see Assenza et al.[148] for their inclusion in an ABM framework.

29 This is despite some empirical work suggesting that consumer confidence contains important information for forecasting personal spending and consumption [158–160].

ical overhaul of the DSGE methodology by uprooting the axiomatic approach that characterises New Keynesian models.

The paradigm shift is radical and the development of new Agent-Based models seems, for the time being, orthogonal to the classical community, both in terms of approach and method.

2.5 AGENT-BASED MODELS AS A NEW PARADIGM TO FOLLOW

Generally speaking, Agent-Based models (ABM), see Refs.[163–166] but also [167, 168] for insights, are a class of computational models aimed at describing large systems/societies/economies starting from the behaviours and interactions of individuals, as in [169]. ABM models claim to be the most viable alternative (in terms of modelling) to classic DSGE models. By construction, ABM models are more versatile than the classic approach, as they are naturally conceived to accommodate many of the missing ingredients that DSGEs (see above) are struggling with.

In particular, the most evident strength of ABMs relies on the fact that they are built starting from a multitude of interacting agents, each following a pre-established decision-making process, making them best suited to grasp the emerging complex behaviour of aggregates, in a "*More is Different*" way[170–172] [30]. ABM models allow for the description of endogenously generated events (dragon kings [176]) which, again, are completely excluded (and ignored) in the DSGE framework. Their presence allows small shocks to amplify via feedback loops, giving rise to a large business cycle, see Ref. [165]. This non-axiomatic freedom might translate into an economy defined by a jagged landscape, where the notion of a global equilibrium is completely lost [177]. In some cases, the external shock constantly shifts the system from one state to the other, describing an economy permanently out of equilibrium. Thanks to numerical simulations, however, the absence of equilibrium can be easily overcome and, thanks to their versatility, ABM models play the role of *generators of possible scenarios*. The introduction of feedbacks and interactions is favoured by the lack of an axiomatic base regulating the rationality of agents. In particular, rational expectations are replaced by different definitions of rationality. Concepts as, *rules of thumb*, see Refs. [178–180], reflect agents' lack of time to compute all the possible scenarios. Thus, their choices are driven by simple estimations of the future. Although rationality still underlies this formulation, the rough estimation provided by rules of thumb allows for systematic errors in the maximisation of future trajectories. In order to deeply

30 However, it is evident how both the DSGE and the ABM communities have worked tirelessly to integrate heterogeneities and networks into the macroeconomic debate, see Refs.[173–175]. This direction is certainly one of the most accepted, although substantial differences remain between the two approaches.

question the axiom of rationality, the ABM community has also imported some concepts from Behavioural Economics, the so-called *behavioural rules*, which include empirically observed behaviours –not completely rational–, such as imitation processes. Within this context, the concept of *bounded rationality*, see [181, 182], is often used to indicate the cognitive limitations of the agent by interpolating them with the knowledge deficits that are inevitable [183]. These ideas are more widely accepted as a fundamental part of human nature and, as such, are implemented in some ABM models [184].

Although Agent-Based models seem to solve all the drawbacks addressed to the DSGE community, they are subject to two major criticisms from the more conservative group of economists.

First, due to the simplicity of the interactions, they have always been referred to as toy models, perhaps sometimes with a pejorative meaning. Their role in the macroeconomic landscape has been underestimated, being suitable only for *proofs of concepts* [185, 186] and not for forecasting purposes.

Second, the large nature of ABMs – both in terms of the number of different agents and their respective rules – combined with results derived primarily from large simulations, has led to resistance in accepting them. Their increasing complexity[186, 187] makes it unfeasible to treat ABM models analytically, and solutions are extracted by numerical simulations. The DSGE [188] community criticises macroeconomic ABMs on the basis that there is no clear analytical link between the parameters, shock inputs and the output aggregate dynamics (many efforts have been made to close this gap see [189, 190]), and may be sensitive to initial conditions [191]. Due to the frequent accusations of lack of transparency and difficulty in evaluation [192], they are also referred to as numerical "black boxes".

In such a sceptical environment, ABM models have evolved considerably in the last years and, thanks to both the increased availability of data[31] and computational power support, they have become capable of reproducing entire economies. To give a perspective, R.Axtel [194] presented a metropolis full-scale quantitative agent-based model meant to study the housing market bubble that triggered the 2008 global financial crisis. S.Poledna [195] et al. built an ABM model of the scale of Austria (see also Ref. [196]), where micro and macro data were used to calibrate agents preferences. More specifically, this model has been used as a forecasting tool to predict medium-run macroeconomic consequences of lockdown measures taken in Austria to limit the spread of Covid-19.

31 Although it is often hard to estimate parameters directly from the data [193].

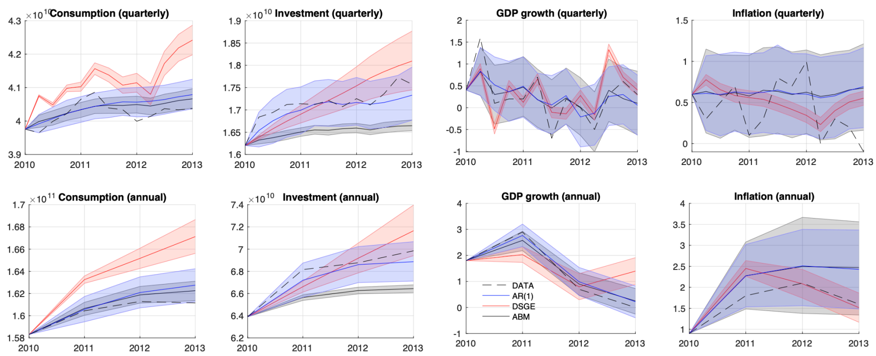

Figure 13: Figure taken from S.Poledna et al. work "*Economic Forecasting with an Agent-Based Model*" [195]. The figure compares real data (black dashed lines) to the performances of DSGE (red lines) and the ABM developed by the author (blue lines). The graph compares the consumption, the investments, the GDP growth and the inflation. Quarterly vs annual comparison is offered between the two rows.

2.5.1 *State of the Art. A Comparison between DSGE and ABM*

Of particular interest is to compare the performance of a sophisticated DSGE model to the outcomes of an ABM with a number of agents comparable, in order of magnitude, to the population of Austria. The work of S.Poledna et al. [195] has caused a stir in the community as it is one of the first ABMs, with forecasting purposes, designed to reproduce the economy of a state. Its structure is the following: first, agents and firms estimate the future state of the economy and make sets choices accordingly. Second, the labour market clears: each firm sets its wages and hires/fires workers based on its estimation of the future productivity. Then, firms ask loans from the central bank and finally are able to produce goods. Market participants are matched, and excesses or shortages can remain. Finally, all the actors change their preferences on realised metrics (realised growth, demand, etc.). The value of most parameters, quite large in number, is calibrated on Austrian 2010 economy. The forecasting performances are then compared with both real data (2010-2013) and with DSGE predictions. The trajectories are shown in Fig. 13.

Strong claims about this ABM outperforming DSGE are hard to formulate, as both frameworks show plausible results. Looking at quarterly consumption and investments, this ABM performs slightly better compared to the DSGE counterpart (real data are included in the error bars), although the error bars are quite broad.

On the other hand, when estimating GDP growth and inflation, the forecasting of DSGE seems to be more accurate, while the trajectory of the ABM is dominated by uncertainties. However, it is useful to note that ABM models are quite new and not yet fully accepted by institutions. Paraphrasing Keynes, is it better to be roughly right about GDP growth or precisely wrong? In that sense, ABM can capture the uncertainty of what can happen, but may not be precise on a quantitative level *circa* what actually happens. The DSGE aims to be quantitatively precise, but also noticeably deviates from the data. In any case, there is not a clear winner, as the two methods are inconclusive for all metrics. The real open question is about understanding the underlying mechanisms of the economy. What is debatable, in fact, are the core hypothesis of the two frameworks.

2.6 BRIDGING THE GAP THROUGH TOY MODELS

The goal of my research is to understand whether, through the addition of some behavioural components (such as feedbacks and heterogeneities), it is possible to *bridge the gap* between the DSGE models and the ABM formulations. This is where Complex Systems and its teachings come into play. In fact, for a statistical physicist, feedback effects and interactions play a dominant role in the phenomenology of systems: both the Ising, see Sec.1.4.2, and the Lennard-Jones models, Sec.1.4.1, perfectly incorporate these two aspects, as none of their results are possible excluding the role of interactions (between first neighbours or among water molecules). Thus, the goal is to show that accounting for those ingredients even the simplest monetary model can display more realistic behaviours.

The work is twofold. Not only is it necessary to build a coherent model – starting from existing ones – but we need to understand which are its fundamental parameters whose variation has macroscopic effects on the economic quantities we study. Generally speaking, we will not seek to build models that are necessarily easy to solve (analytically) because in such scenario we can rely on the computational analysis, which – in this research – will play the role of "*telescope of the mind*" [165]. Just as the introduction of the telescope during the 17th century allowed to see places that were previously inaccessible, exploiting computer simulations, we will be able to systematically explore the parameters' space of our models identifying the economically relevant quantities.

For the sake of clarity, we aim to keep the formulation of our setups as simple as possible, limiting the number of parameters to a minimum. The model from which my analysis starts, presented in Ch.3, can be defined in all respects as a *toy model*. Its apparent simplicity is not matched by the sophistication of its results. Once the toy model is stripped of all doubts, it provides an excellent starting point for adding new ingredients. Toy models are the "theatre of reality": through them, the complexity of the economy can be decomposed into its most salient aspects, which are studied individually. Once we are satisfied with the plot, the whole *pièce* can be written and played. If toy models are based on reasonable assumptions, however simple, they can provide general schematic results.

To further support the use of toy models, physics teaches us that many microscopic details are not fundamental in order to establish a correct view of reality. The Ising's model is not microscopically accurate (a magnet is not a set of binary variables with only nearest neighbours interactions), yet it manages to predict the paramagnetic/ferromagnetic transition. Similarly, the Lennard-Jones formulation describes water molecules as rigid spheres, omitting many details, but its results are consistent with experience.

Our approach departs radically from the more complex models presented above. Instead of, as in meteorology, focusing on trajectories and their relative error, we will analyse macroeconomic models by making plausible microscopic assumptions, studying how these changes add possible stylised scenarios in the dynamics of aggregate quantities. Among these effects we will discuss in particular the presence of economic crises of different sizes and duration, and the existence of phase diagrams and phase transitions.

A classical monetary model: Highlights

This chapter has a double function. First, it introduces the basic DSGE model on which the second part of the thesis is based, together with a stylised version of a HANK model. This helps the reader to form an intuition of how heterogeneities are taken into account in New Keynesian models. The second part of the chapter is dedicated to a comparison of the DSGE and the ABM approaches. The aim is to highlight the criticalities of both frameworks. In particular, the three main interconnected strands are: general equilibrium, exogenous and endogenous shocks, the representative agent paradigm and finally the concept of rationality.

Part III

FEEDBACK MONETARY MODELS

This second part of this document contains the results of my personal research, that I have conducted (together with my Ph.D. directors) during the past three years. The structure is as follows: the first chapter, Ch.3, deals with a simple but extremely rich modification of the basic DSGE. We will explore how by introducing a feedback effect the dynamics of aggregate macroeconomic quantities is completely overturned. This is also the starting point for the following chapters. The following chapter, Ch.4, concerns a heterogeneous extension of the previous chapter. We will see how heterogeneities play a fundamental role in the economy, and compare the results to real data. Finally, in the last chapter, Ch.5, we will introduce the notion of capital and, with it, an indicator that will make agents' investments lean towards the bond market or rather towards the capital market.

3 CONFIDENCE COLLAPSE IN A DSGE MODEL

The starting point of this chapter is the simplest possible DSGE model, as presented in section Sec.2.2. In Ch.2 we showed how the total absence of feedback effects in the benchmark formulation of New Keynesian models results in the absence of economic crises. As broadly discussed, the fluctuations of the aggregate quantities of the benchmark model are purely exogenous, when, on the contrary, the underlying reasons of the Global Financial Crisis are nowadays credited to phenomena whose nature is, instead, endogenous. Many ingredients that were missing in previous versions of the model (such as the absence of a financial sector) have been added in the recent years, in an attempt to assuage some of the scathing criticisms that were uttered post GFC (see for example [85, 87–89] and [72, 90, 93] for rebuttals).

However, the whole DSGE framework seems to be – partly for technical reasons – wedded to the Representative Agent paradigm, and to a (log-)linear approximation scheme that describes small perturbations away from a fundamentally stable stationary state. Economical recessions are therefore difficult to accommodate within the scope of DSGE.

Ignoring the effects of feedbacks and interactions in the DSGE framework, as it happens to the ideal gas laws, has resulted in the inability of these models to illustrate all emergent properties, hiding important phenomena such as phase transitions. On the other hand agent-based models are by construction very adapted to account for heterogeneities and interactions[165, 176, 197–199]. This can generate non-linear effects that lead to different regimes, where crises are naturally included in the model. For other examples on how to integrate economic recessions into models, see Refs. [200, 201].

In order to bridge the gap between DSGE and ABMs and allow interesting non-linear phenomena, such as trust collapse, to occur within DSGE, in this model we replace the representative household by a collection of homogeneous but interacting households.

The interaction is introduced as each household forms its sentiment, i.e. propensity to consume, by observing what the average consumption of its neighbours was during the previous time period. The mechanism is intended to reproduce those panic effects underlying the GFC: when an agent observes that the consumption of its neighbours has dropped, it will lower its propensity to consume, as a form of precautionary savings, see examples [202–204]. In a certain sense, this is like, *mutatis mutandis*, going from the law of perfect gas, with a unique thermodynamic equilibrium around which only small fluctuations are possible, to the Van der Walls equation of states, where the feedback on the pressure due to attractive interaction is taken into account opening up the possibility of the liquid gas phase transition. Similarly, in our model, the addition of the confidence feedback on the propensity to consume opens up the possibility that a relatively small decline in the overall production will lead to a collapse in confidence and a sharp drop in the economic activity. The existence of two different equilibria has also been recently suggested in another context in [205].

Following the standard procedures of statistical physics, we will draw the phase diagram of our model. On this representation we identify the regions where bi-stability and economic crises occur. If the effect of feedback is small enough, our model will be equivalent, albeit with increased volatility, to the starting DSGE model. On the other hand, when the strength of the feedback increases, we progressively enter a phase in which the economy admits economic crises, during which output and consumption collapse abruptly but remain short-lived. When the role of feedback grows further, technological shocks can induce persistent economic crises. In such regime the presence of such economic crises occurs even when the noise level is very low, in a *small shocks large business cycle* fashion.

In particular, one of the most important findings of this chapter is that the probability of such crises occurring is exponentially dependent on the parameters and therefore, a bad calibration can lead to completely misleading results. This model is therefore an example of, *unknown-knows*. In the economy here described, economical recessions are a possible state of the world, but their probability is fundamentally hidden by the impossibility of calibrating the parameters with absolute certainty.

The work presented here relates to various strands of the literature that emphasises the role of multiple equilibria and self-fulfilling prophecies, in particular the work of Brock & Durlauf on social interactions [206]. Technically, the model presented here is similar to the literature on "habit formation" or "keeping up with the Joneses" (KUJ) [207–209], although the mechanism we introduce would be better transliterated as "keeping down with the Joneses" (KDJ). In fact, the social pressure to consume here is replaced by trust, a lack of which cuts the consumption. Even if we are more concerned about the self-reflexive collapse of confidence than about consumption surges, these are still described by one of the stages of the model. Other works, such as Cooper et al. [210] study coordination failures in Keynesian model. However, in their work the multiple equilibria are not dynamics and once chosen the system doesn't depart from it. In the following, we introduce a model where the system jumps from different steady state, with probability that depends on parameters. Other examples are the work of Angeletos et al. [211] where they study confidence mechanisms within firms, albeit in a single equilibrium framework. Confidence is there introduced as an exogenous random variable, which is responsible of miss coordination between firms. In our model, confidence is a quenched parameter that can be affected by policy measures.

In the literature, several other scenarii can lead to the coexistence of static equilibria, corresponding to high/low confidence. S [111, 212, 213], high/low output [165, 214, 215], or high/low inflation expectations [145, 214], trending/mean-reverting markets [216–219], etc. with possible sudden shifts between the two. Multiple equilibria can be either a result of learning from past events, or from strong interactions between individual agents (direct or mediated by markets) – for a review, see e.g. [41]. Another, distinct line of research explores the consequences of having an indeterminate equilibrium, i.e. a stationary solution around which small fluctuations can develop without being pinned by initial conditions (for a recent review and references, see [220, 221]). These fluctuations are not related to any real economic driving force, but rather the result of self-fulfilling prophecies. In our present model, fluctuations *are* triggered by real technology shocks, but are then amplified by a self-reflexive mechanism. Nothing would prevent, however, the existence of further indeterminacy around different stationary points.

3.1 A MULTI-HOUSEHOLD DSGE MODEL

As many of the algebraic steps have been explicitly written in the previous chapter, they are deliberately omitted here as they would result redundant. For the sake of clarity, the model's core equations are highlighted also with the purpose to show the reader how they are modified by new assumptions.

3.1.1 *The Households*

As mentioned in the introduction, this model has as a starting point the benchmark DSGE model presented in Sec.2.2. Here, we model M identical households, all described by the same utility function (as in the HANK formulation). Similarly to Eq.(115), we assume that each household $i \in [\![1, M]\!]$ is characterised by a utility function $U_i(c_t^i, n_t^i)$ that depends on its (unique good) consumption c_t^i and amount of labour n_t^i as:

$$U_i(c_t^i, n_t^i) = f_t^i \frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \gamma_i \frac{(n_t^i)^{1+\phi}}{1+\phi}\ , \tag{63}$$

where γ_i is a factor measuring the disutility of labour, and $\sigma \in]0, 1]$ and $\phi > 0$ are two i-independent parameters allowing to get the correct concavity. Following the standard choices we set $\sigma = \phi = 1$. The resulting log-utility reads:

$$U_i(c_t^i, n_t^i) = f_t^i \log c_t^i - \gamma_i \frac{1}{2}(n_t^i)^2 \,. \tag{64}$$

The quantity f_t^i is a time-dependent factor measuring the confidence of the household i at time t, and hence their propensity to consume. This "belief function" [221] will be responsible for the possible crises in this model.

Each infinitely lived household maximises its future expected discounted utility with a discount factor $\beta < 1$, subject to the budget constraint [78]:

$$p_t c_t^i + \frac{b_t^i}{1+r_t} \leqslant w_t n_t^i + b_{t-1}^i + E_t^i, \tag{65}$$

where p_t is the price of the good, w_t the nominal wage (assumed to be identical for all households), and E_t^i any extra source of income (dividends, subsidies, taxes). b_i^t the amount of bonds paying 1 at time $t+1$, purchased at the time t at price $(1+r_t)^{-1}$, where r_t is the interest rate (set by the central bank). The maximisation is achieved using the standard Lagrange multipliers method over the quantities c_t^i, n_t^i and b_t^i. This gives the household's state equations, Eqs.(66), (67) and the Euler equation, Eq.(68):[1]

$$\begin{aligned}
\left(c_t^i\right)^\sigma &= \frac{f_t^i}{\lambda_t^i p_t} && (66)\\
\left(n_t^i\right)^\phi &= \omega_t p_t \frac{\lambda_t^i}{\gamma_i} && (67)\\
f_t^i \left(c_t^i\right)^{-\sigma} &= \beta(1+r_t)\mathbb{E}_t\left[\frac{f_{t+1}^i \left(c_{t+1}^i\right)^{-\sigma}}{1+\pi_{t+1}}\right], && (68)
\end{aligned}$$

where, coherently with the notation of Sec.2, $\omega_t = w_t/p_t$ is the real wage, $\pi_t := p_t/p_{t-1} - 1$ the inflation rate and λ_t^i a Lagrange multiplier. We define the aggregate consumption as $C_t := \sum_{i=1}^M c_t^i$ and the total amount of labour provided by the households to the firm as $N_t := \sum_{i=1}^M n_t^i$.

3.1.2 *The Representative Firm*

The unique firm has a technology z_t such that at each period the production Y_t is given a Cobb-Douglas production function:

$$Y_t = M^\alpha z_t \frac{N_t^{1-\alpha}}{1-\alpha}, \tag{69}$$

1 Although the Euler equation will actually be irrelevant for most of our story, we will use it in the last section when we provide an approximate calculation of inflation in the presence of confidence effects.

where z_t is the total factor productivity, and α is set to $1/3$, as in the benchmark model. Differently from Eq.(21), the output of the firm is multiplied by a scaling factor M^α. This term assures the correct limit in the case where $M \to \infty$; allowing both the total production and the aggregate consumption to be proportional to the number of households M. As in the reference model, Eq.(22), the technology $z_t = \bar{z}e^{\xi_t}$ is log-normal distributed and the log-productivity ξ_t follows an AR(1) process:

$$\xi_t = \eta_z \xi_{t-1} + \sqrt{1-\eta_z^2}\,\mathcal{N}\left(0, \sigma_z^2\right) \ , \tag{70}$$

where η_z modulates the temporal correlations of the technology shocks, and σ_z the amplitude of these shocks.

Each period, the firm maximises its profit with the assumption that markets will clear, i.e. that $Y_t = C_t$. The real profit function is given by $\mathbb{P}_t/p_t = C_t - \omega_t N_t$. Its maximisation with respect to the aggregate labour N_t, yields to $\omega_t = z_t(M/N_t)^\alpha$, i.e., the firm hires labour up to the point where its marginal profit equals the real wage [78]. Now, assuming for simplicity that f_t^i and γ_i are all equal (homogeneous beliefs and preferences) leads to $c_t^i = c_t = C_t/M$, $n_t^i = n_t = N_t/M$, $\gamma_i = \gamma$ and $f_t^i = f_t$. Exploiting Eqs. (66,67) and Eq. (69) with $Y_t = C_t$ it is possible to find the solution for c_t, n_t and ω_t as a function of f_t and z_t. we set, here and for the rest of the chapter, the value of the exponents to $\phi = \sigma = 1$, $\alpha = 1/3$. This yields to:

$$c_t = z_t\left(\frac{9f_t}{4\gamma}\right)^{1/3} \tag{71}$$

$$n_t = \left(\frac{2f_t}{3\gamma}\right)^{1/2} \tag{72}$$

$$\omega_t = z_t\left(\frac{3\gamma}{2f_t}\right)^{1/6} . \tag{73}$$

The next step is to determine the shape of the feedback f_t.

One point deserves to be commented. This extremely simplified setting leads to the following situation: assuming the interest rate as a constant, the bonds' dynamics is set by the budget constraint and becomes trivial. We have $b_t = (1+r)(b_{t-1} - \mathbb{P}_t/M)$. After its maximisation, and given the solutions provided by Eqs. (71, 72, 73) the profit function $\mathbb{P}_t/M \sim 0.44 z_t p_t (f_t/\gamma)^{1/3}$ is strictly positive. Therefore, at large t, b_t diverges and its sign depends on the value of b_0 and r itself. To this regard, this toy model is in a certain sense in a *Ponzi* condition. However, there is one important aspect to take into account in a more sophisticated DSGE framework, the firm's profits are re-distributed under the form of dividends among the households. The lack of this mechanism, on the one hand, keeps the model as simple as possible, on the other hand it allows for the presence of losses. To account for this we have added a term E_t^i in the budget constraint, Eq.(65) representing the additional sources of income the households have which is not taken into account in this model. This issue recurs in the following chapters, so the origin of the term E_t^i must be explained in detail.

3.2 ANIMAL SPIRITS AND SELF-REFLEXIVITY

Now, the main innovation introduced in this chapter is to assume that the sentiment of households at time t (which impacts their consumption propensity f_t) is a function of the past realised consumption *of others*, itself revealed by a consumer sentiment index. If the household i observes that other households have reduced their consumption in the previous time step, it interprets it as a sign that the economy may be degrading [202–204]. This increases its precautionary savings and reduces its consumption propensity. Conversely, when other households have increased their consumption, confidence of a household i increases, together with its consumption propensity. A general specification for this animal spirits feedback is

$$f_t^i \longrightarrow F\left(\sum_{j=1, j\neq i}^{M} J_{ij} c_{t-1}^j\right), \tag{74}$$

where $F(\cdot)$ is a monotonic, increasing function and J_{ij} weighs the influence of the past consumption of the household j on the confidence level of i. In this work, we consider the case of a fully connected network, i.e. $J_{ij} = J/M$, i.e. only the consumption "index" matters. We will furthermore consider the large M limit such that:

$$\frac{J}{M}\sum_{j=1, j\neq i}^{M} c_{t-1}^j \to J c_{t-1} \; .$$

This corresponds to a *mean field* approximation in statistical physics. While it neglects local network effects, it captures the gist of the mechanism we want to illustrate and furthermore allows us to keep the household homogeneity assumption (different local neighbourhoods generally lead to different consumption propensities). Some anticipations: in the next chapter, Ch.4 we study the case where the interaction network is sparse, and heterogeneities dominate the dynamics.

Combining (128) and (74) yields to the solution of this extended DSGE model, which reads:

$$c_t = e^{\xi_t} G(c_{t-1}), \quad \text{with} \quad G(x) := \bar{z}\left(\frac{9F(x)}{4\gamma}\right)^{1/3} . \tag{75}$$

Equation (75) is a discrete time evolution equation for the consumption level. In order to exhibit how this dynamics can generate excess

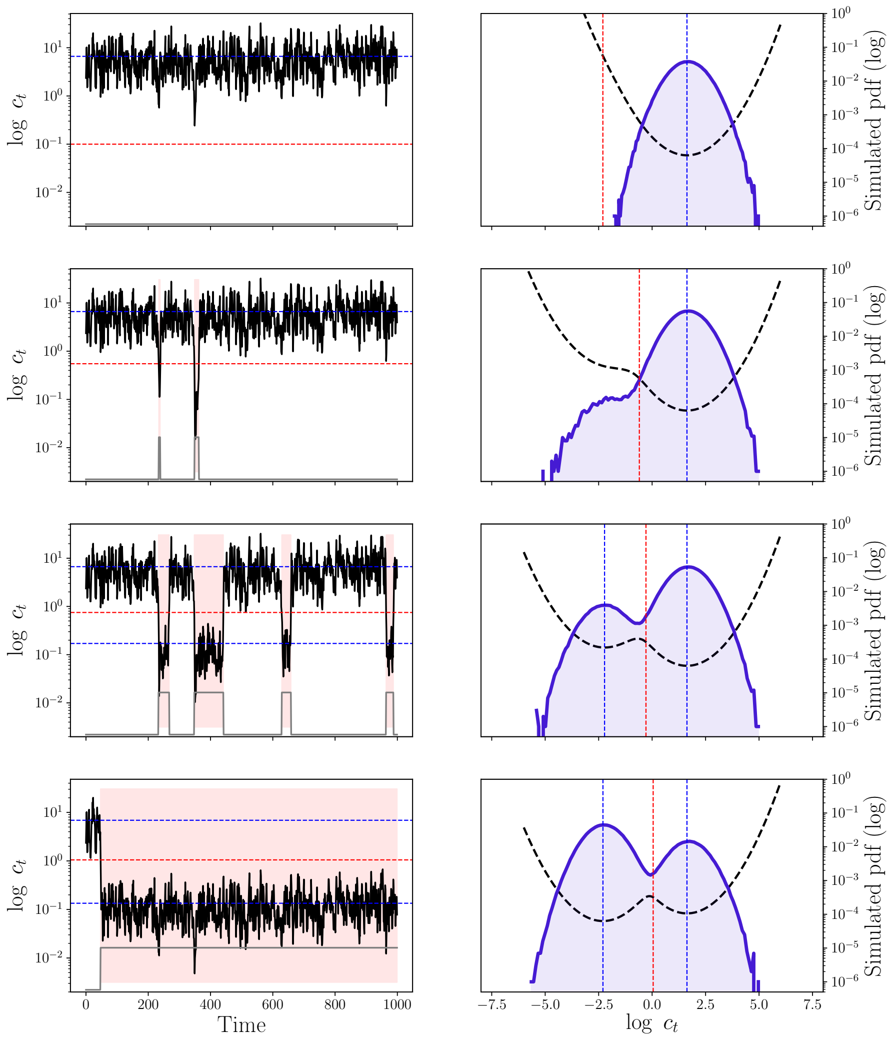

Figure 14: Numerical simulation of the model for increasing values of the confidence threshold c_0 and for fixed values of $\theta = 5$, $\sigma_z = 0.6$ and $\eta_z = 0.5$. Left column: temporal trajectories of the log output $\log c_t$ with a horizontal dot-dashed red line located at $\log c_0$ and dashed black lines at $\log c_{>,<}$; right column: (log-)probability distribution $p(x)$ of the output, with the corresponding positions of $\log c_0$ and $\log c_{>,<}$. From up to bottom: $c_0 = 0.1$ (A phase, no crises, Gaussian distribution of output); $c_0 = 0.55$ (B^+ phase, short crises, increased volatility and skewed distribution of output); $c_0 = 0.75$ (C phase, long recessions, bi-modal distribution with most weight on $\log c_>$); $c_0 = 1.05$ (C phase, long recessions, bi-modal distribution with most weight on $\log c_<$). Dashed lines: effective potential $2V(x)/\sigma^2$, defined in Sec. 3.4.

volatility and endogenous crises, we assume that $G(x)$ is a shifted logistic function (but any S-shaped function would lead to qualitatively similar results).[2] To wit:

$$G(c) = \frac{1}{2}\left((c_{max} - c_{min})\tanh(\theta(c - c_0)) + c_{max} + c_{min}\right) , \qquad (76)$$

where c_{min}, c_{max}, c_0 and θ are parameters with the following interpretation:

- $c_{min} > 0$ is the minimum level of goods that households will ever consume when productivity is normal (i.e. $\xi_t = 0$).
- $c_{max} > c_{min}$ is the maximum level of goods that households will ever consume when productivity is normal (i.e. $\xi_t = 0$).
- c_0 is a *confidence threshold*, where the concavity of $G(c)$ changes. Intuitively, $c > c_0$ tends to favour a high confidence state and $c < c_0$ a low confidence state.
- $\theta > 0$ sets the width over which the transition from low confidence to high confidence takes place: in the limit $\theta \to +\infty$, one has $G(c < c_0) = c_{min}$ and $G(c > c_0) = c_{max}$.[3]

The standard DSGE model, where the animal spirit feedback is absent, is recovered in the limit $\theta c_0 \to -\infty$, in which case $G(c) \to c_{max} = \text{cst}$. The dynamics of our extended model falls into four possible phases, that we will call A, B^+, C and B^- (see Figs. 14, 16), and discuss their properties in turn. In the following, we will use the notation $\Delta := c_{max} - c_{min}$.

3.3 PHASE DIAGRAM

The phase diagram is drawn comparing the RHS and LHS (a line of angular coefficient 1) of the Eq.(75). In the absence of external noise ($\xi_t = 0$) and with θ fixed, five scenarios are possible. First, if the centre c_0 of the RHS is small, for all possible values of θ it exists only one intersection between RHS and LHS, noted $c_> \sim c_{max}$, and the equilibrium is unique. As the confidence threshold c_0 increases, one encounters the tipping case where exactly two solutions exist. This intermediate case happens when the point of the RHS having derivative equal to unity lies on the line representing the LHS. As the feedback function is S-shaped, the situation in which there are exactly two solutions occurs twice, as it is schematically shown in Fig.15. Between these two limit values of c_0 there are three points of

2 Whereas the existence of feedback is clear [202–204], we are not aware of studies attempting to quantitatively measure the function G. The change of concavity however seems plausible from a behavioural viewpoint.

3 In this chapter we fix θ and vary c_0. Note however that fixing c_0 and varying the "temperature" θ would also be of interest to investigate the effect of population heterogeneity. This scenario is addressed in the next chapter.

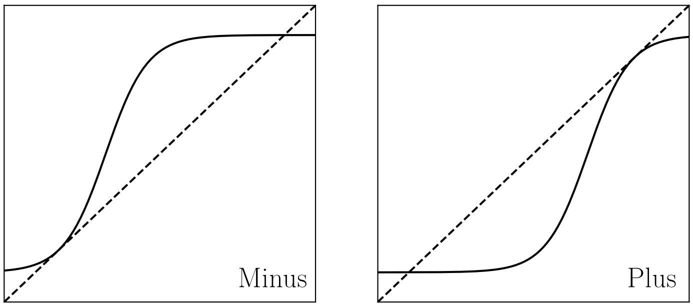

Figure 15: The figure shows the two limit case delimiting the phase C and corresponding to the situation where the point of the RHS having derivative equal to unity lies on the line representing the LHS of Eq.75. The labels "Minus" and "Plus" correspond to the choice of sign in the Eq.(77)

intersection between the RHS and the LHS. If one increases further the confidence threshold, the solution is again unique and of the same magnitude as $c_< \sim c_{\min}$. In the light of this discussion, it is clear that the critical pairs of c_0 and θ, noted c_0^* and θ^* respectively, correspond to the pairs for which there are exactly two solutions. Setting $\upsilon = \frac{\Delta\theta^*}{2}$, these can be derived analytically and correspond to all pairs (c_0^*,θ^*) that satisfy the following relation:

$$c_0^* = c_{\min} + \frac{\Delta f_\pm(\upsilon)}{f_\pm(\upsilon)+1} - \frac{1}{\theta^*}\log f_\pm(\upsilon)\,; \quad f_\pm(\upsilon) = \left(\upsilon^{1/2} \pm (\upsilon-1)^{1/2}\right)^2 . \tag{77}$$

The "Minus" case of Eq.(77) defines the C/B^+ critical line. *Vice versa* the "Plus" case corresponds to the C/B^- phase line, see again Fig.15 for an intuitive scheme.

To separate phases A and B^+ we need to account for the noise, i.e. $\xi_t \neq 0$. The phase A correspond to the couples (c_0, θ), for which any given realisation of the noise (that multiplies the RHS) can introduce the three solutions scenario. The relative position of the boundary of the A phase therefore depends on whether $\theta\Delta$ is larger or smaller than 2. In the first case – corresponding to Fig. 16, the boundary is with the B^+ phase (to be described below). In the (c_0, θ) plane, the A phase is located on the left of the hyperbola defined by

$$\theta c_0 = 1 + \frac{2c_{\min}}{\Delta}. \tag{78}$$

In the case $\theta\Delta < 1$, the boundary is with the B^- phase and the A phase corresponds to $c_0 \leqslant c_{\min} + \Delta/2$. To this point, the four phases of this model are geometrically identified, but it remains to develop an intuition of what happens within them.

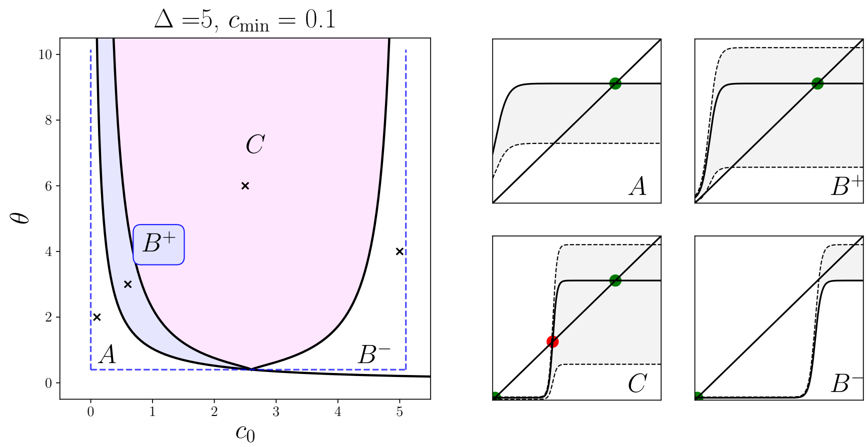

Figure 16: Left: Phase diagram of the model, with analytically determined boundaries. Phase A: High Output, No Crises; Phase B^+: High Output with Short-Lived Recessions; Phase C: Long-Lived Booms & Recessions; Phase B^-: Phase B^-: Low Output with Short-Lived Spikes. Right: Graphical representation of the iteration $c_{t+1} = e^{\xi_t} G(c_t)$ in the different phases. The plain line corresponds to $\xi_t = 0$. Clearly, any S-shaped function G would lead to similar effects.

3.3.1 *Phase* A: *High Output, No Crises*

This phase corresponds to the DSGE phenomenology, where the equilibrium is unique and the only solution of $e^{\xi} G(c) = c$ is a high consumption solution $c > c_0$ for all values of ξ. Even for large negative shocks $\xi < 0$, the economy remains in its confident state. For small noise amplitude $\sigma_z \ll 1$, the consumption remains around the value $c_>$ solution of $G(c_>) = c_>$, and one can linearise the dynamics around that point:

$$\delta_{t+1} \approx G'(c_>)\delta_t + \xi_t, \qquad \delta_t := \frac{c_t - c_>}{c_>}. \tag{79}$$

This leads to the following expression for the consumption volatility:

$$\mathbb{V}[\delta] = \frac{\sigma^2}{1 - G_>'^2} \frac{1 + \eta G_>'}{1 - \eta G_>'}, \qquad G_>' := G'(c_>). \tag{80}$$

In other words, the output volatility is proportional to the amplitude σ_z of the technology shocks – small shocks lead to small volatility (note that $G_>' < 1$ in the whole A phase). However, the feedback mechanism leads to *excess volatility*, since as soon as $G_>' > 0$, one has

$$\frac{\sigma_z^2}{1 - G_>'^2} \frac{1 + \eta_z G_>'}{1 - \eta_z G_>'} > \sigma_z^2. \tag{81}$$

3.3.2 *Phase* B^+*: High Output with Short-Lived Recessions*

In this phase, B^+, there is still a unique equilibrium state when productivity is normal, i.e. a unique solution to $G(c_>) = c_>$ with $c_> > c_0$. However, downward fluctuations of productivity can be strong enough to give birth to two more solutions $c_< < c^* < c_0$, one unstable (c^*) and one stable ($c_<$). With some exponentially small probability, when $\sigma \to 0$ (see (82) below), the economy can be driven out of the normal state $c_>$ and crash into a low output state, in which it will remain trapped for a time of the order of $T_\eta := -1/\log(\eta_z)$, i.e. the auto-correlation time of ξ_t. In other words, sufficiently large fluctuations of output are initially triggered by a relatively mild drop of productivity, which is then amplified by the self-referential "panic" effect. But since the low output state is only a transient fixed point, the recession is only short-lived.

3.3.3 *Phase* C*: Long-Lived Booms and Recessions*

Phase C is such that the equation $G(c) = c$ has two stable solutions $c_<, c_>$ and one unstable solution c^*. This phase is delimited, in the (c_0, θ) plane, by a parabolic boundary (see Fig. 16) with $c_0 \to (c_{\min} + c_{\max})/2$ when $\theta\Delta \to 2^+$ and $c_0 \to c_{\min}$ or $c_{\max}$ when $\theta \to \infty$. The lower boundary $C \to B^+$ corresponds to $c_< \to c^*$ before both disappear, leaving $c_>$ as the only solution, whereas the upper boundary $C \to B^-$ corresponds to $c_> \to c^*$ before both disappear, leaving now $c_<$ as the only solution.

In the absence of fluctuations ($\sigma_z = 0$), the economy in phase C settles either in a low output state or in a high output state. But any, however small, amount of productivity fluctuations can induce transitions between these two states. The time needed for such transitions to take place is however *exponentially long* when $\sigma_z \to 0$:

$$\log T(c_{>,<} \to c_{<,>}) \quad = \quad \frac{W(c_{>,<} \to c_{<,>})}{\sigma_z^2} + O(\sigma_z^0); \qquad (82)$$

where $W(c_> \to c_<)$ and $W(c_< \to c_>)$ are computable quantities (see section 3.4 below and Fig. 17). This is clearly the most interesting regime: the economy can remain for a very long time in a high output state $c_>$, with relatively mild fluctuations (in fact still given by Eq. (80)), until a self-fulfilling panic mechanism throws the economy in a crisis state where output is low ($c_<$). This occurs with a Poisson rate $1/T(c_> \to c_<)$. Unless some explicit policy is put in place to restore confidence, the output will linger around $c_<$ for a Poisson time $\sim T(c_< \to c_>)$ which is also very long when $\sigma_z \to 0$.[4] Note

4 Note in particular that $T(c_< \to c_>)$ is much longer than T_η: it is no longer the correlation time of productivity fluctuations that sets the duration of recessions (at variance with the B^+ scenario).

that $T(c_> \to c_<)$ is the average time the system remains around $c_>$ before jumping to $c_<$. The actual time needed to transit is itself short, and the resulting dynamics is made of jumps between plateaus – see Fig. 14. A downward jump therefore looks very much like a "crisis".

As we discuss below, recession durations are much shorter than the time between successive crisis when $c^* - c_< < c_> - c^*$, i.e. when the low output solution is close to the unstable solution, which plays the role of an escape point. As c_0 grows larger, still remaining within the C phase, one will eventually be in a situation where $c^* - c_< > c_> - c^*$, in which case recession periods are much longer than boom periods. As σ_z grows larger, the output flip-flops between $c_<$ and $c_>$ at an increasingly faster rate, see Fig. 17. While it becomes more and more difficult to distinguish crisis periods from normal periods, the output volatility is dramatically amplified by the confidence feedback loop.

3.3.4 *Phase* B^-*: Low Output with Short-Lived Spikes*

Phase B^- is the counterpart of phase B^+ when c_0 is to the right of the phase boundary. In this case, the only solution to $G(c) = c$ is $c_<$: confidence is most of the time low, with occasional output spikes when productivity fluctuates upwards. These output peaks are however short-lived, and again fixed by the correlation time T_η.[5]

3.3.5 *Remarks*

Although quite parsimonious, this model is rich enough to generate a variety of realistic dynamical behaviour, including short-lived downturns and more prolonged recessions, see Fig. 14. We tend to believe that the most interesting region of the phase space is in the vicinity of the B^+/C boundary, and that the 2008 GFC could correspond to a confidence collapse modelled by a sudden $c_> \to c_<$ transition.[6] The behaviour of the economy in the B^- phase, on the other hand, does not seem to correspond to a realistic situation. One of our major result is that the crisis probability is exponentially sensitive to the parameters of the model.

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5 Note that there is no "A^-" analogue of the A phase described above – this is due to the fact that the productivity factor z_t can have unbounded upwards fluctuations but cannot become negative.

6 The role of trust in the unravelling of the 2008 crisis is emphasised in Bernanke, B., Geithner, T. F., & Paulson, H. M. (2019). Firefighting: The financial crisis and its lessons. Penguin Books.

3.4 A THEORY FOR TRANSITION RATES

3.4.1 *Discrete Maps*

Let us now discuss in more detail one of the most important predictions of our model, namely the exponential sensitivity to σ_z of the crisis probability, Eq. (82). Such a result can be obtained by adapting the formalism of [222] to the present problem. In terms of $x_t := \log c_t$, the map (75) reads:

$$x_t = H(x_{t-1}) + \xi_t, \tag{83}$$

with $H(x) := \log G(e^x)$. In the limit of white noise (i.e. $\eta_z = 0$ in Eq. (70)), this is precisely the general problem studied in [222] in the case where $H(x) = x$ has two stable solutions and an unstable one in-between. The authors show that the average time before jumping from one stable solution to another is given, for smalls σ_z, by Eq. (82). They provide an explicit scheme to compute (at least numerically) the quantity W, called the *activation barrier* in physics and chemistry. The idea is to find the most probable configuration of ξ_t's that allows the system to move from one stable position to another. In a nutshell, this amounts to finding a heteroclinic connection, in an enlarged space, between the starting point and the intermediate, unstable fixed point $x^* = \log c^*$ [223].

It is straightforward to generalise the approach of [222] and see that the jump rate has the same exponential dependence on σ_z^2 when the correlation time T_η is non-zero, as confirmed by Fig 17. However, finding the value of W is more complicated. Approximation methods can be devised in the continuous time limit, that we describe now.

3.4.2 *Continuous Time Limit*

Let us slightly change the dynamics by assuming that x_t depends not on the previous value x_{t-1} but rather on an exponential moving average $\bar{x}_{t-1}$ of past values of x, defined recursively as

$$\bar{x}_{t-1} = (1-\epsilon)\bar{x}_{t-2} + \epsilon x_{t-1} \tag{84}$$

Eq. (83) instead reads $x_t = H(\bar{x}_{t-1}) + \xi_t$. Eliminating x_t yields

$$\bar{x}_t - \bar{x}_{t-1} = \epsilon(H(\bar{x}_{t-1}) - \bar{x}_{t-1} + \xi_t), \tag{85}$$

In the limit $\epsilon \to 0$, this equation becomes a Langevin (or SDE) equation for $\bar{x}_t$, for which a host of results is available. It is useful to introduce a potential function $V(x)$ such that $V'(x) = x - H(x)$.

The potential $V(x)$ has two minima "valleys" corresponding to $\log c_<$ and $\log c_>$ and a maximum "hill" corresponding to $\log c^*$ (see rightmost panels of Fig. 14). With this representation, the dynamics

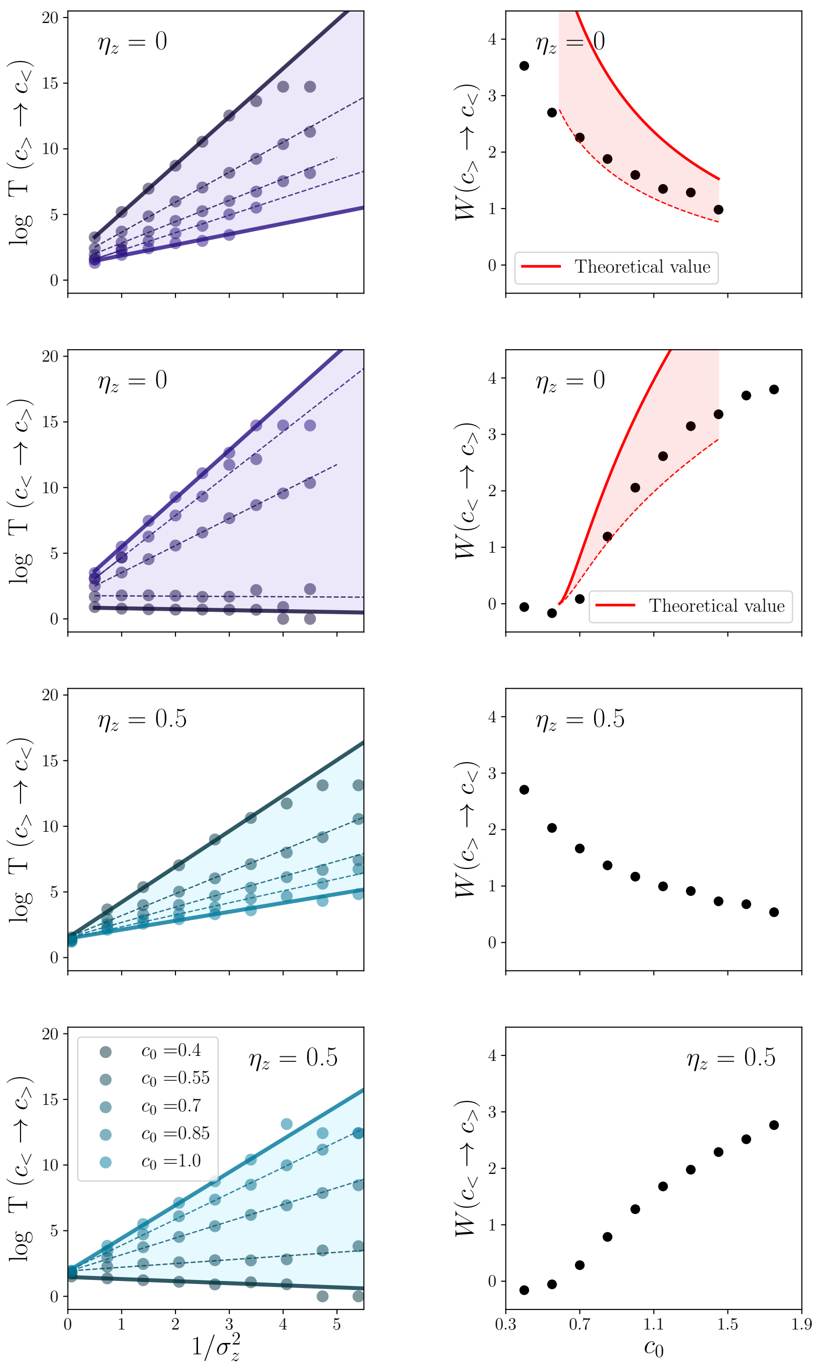

Figure 17: Plot of $\log T(c_> \to c_<)$ and $\log T(c_< \to c_>)$ (left column) vs σ_z^{-2} for different values c_0, and $\eta = 0$ (upper panels) and $\eta_z = 0.5$ (lower panels). The value of c_0 increases with the point's tonality becoming darker. The linear dependence confirms the validity of (82). The right column shows the corresponding barriers W as a function of c_0. For $\eta_z = 0$, we plot the continuous time prediction (86) with $\varepsilon = 1$ (solid red), which overestimates the true barriers (dotted red) by a factor ≈ 2.

of $\bar{x}_t$ under Eq. (85) becomes transparent: for long stretches of time, $\bar{x}_t$ fluctuates around either $x_< = \log c_<$ or $x_> = \log c_>$, until rare fluctuations of ξ_t allow the system to cross the barrier between the two valleys. Calculating the rate Γ of these rare events is the classic problem *Kramers problem* (for a comprehensive review, see [224]). In the limit $T_\eta = 0$ where the noise is white, the final exact expression is, for $\sigma_z \to 0$:

$$\begin{aligned}\Gamma(x_> \to x_<) &= \frac{\sqrt{|H'(x_>)H'(x^*)|}}{2\pi} \exp\left(-\frac{2W}{\varepsilon\sigma_z^2}\right), \\ W &:= V(x^*) - V(x_>), \end{aligned} \tag{86}$$

and *mutatis mutandis* for $\Gamma(x_< \to x_>)$. Such a prediction is compared with numerical simulations in Fig. 17; it overestimates the real barrier by a factor ≈ 2. The most important feature is the exponential dependence of this rate on the height of the barrier W and on the inverse noise variance σ_z^2.[7]

3.4.3 *Exponential Dependence and "Unknown Knowns"*

It is worth emphasising the economic consequences of this exponential dependence of the probability of crises in our model. Clearly, any small uncertainty about the parameters of the model (i.e. $c_0, c_{min}, c_{max}, \theta$) or for that matter, the precise specification of the function $G(c)$, or any other feature neglected in the model, will no doubt affect the precise value of the barrier W. But in the rare event regime $W/\sigma_z^2 \gg 1$, any uncertainty on W is exponentially amplified. Take for example $W/\sigma_z^2 = 25$; a small relative error of 10% on W changes the crisis rate by one order of magnitude. Precisely as the famous butterfly effect (i.e. the exponential sensitivity to initial conditions) forbids any deterministic description of chaotic systems, the exponential dependence of the crisis rate means that this rate is, for all practical purposes, unknowable. Since the probability of rare events cannot be determined empirically, it means that no market can provide a rational valuation of the corresponding risks. This is an interesting example of "unknown knowns", where what may happen is known, but its probability impossible to quantify and cannot be priced.

3.5 INFLATION AND NARRATIVE-BASED MONETARY POLICY

One of the major difficulties in introducing, as in the benchmark model Sec.2.2, a monetary policy (Central bank) is the presence of multiple equilibria. This doesn't allow one to linearise the macroeconomic quantities and the term $\mathbb{E}_t\left[f^i_{t+1}(c^i_{t+1})^{-\sigma}/(1+\pi_{t+1})\right]$ appear-

7 The generalisation of Kramers' result for so-called coloured noise (i.e. $T_\eta > 0$) is also available, see [225]. In this case, corrections to Eq. (86) can be systematically computed, but the exponential dependence of Γ on σ_z^{-2} is preserved.

ing in the Euler equation, see Eq.(68), remains hard to estimate. This makes the task of calculating inflation difficult, and impossible in the phases where the jumps are frequent. Let then first assume that the crisis probability is very small, so one can linearise the Euler equation, Eq. (68) with $\sigma = 1$.

In the absence of frictions, the model is usually closed by assuming a Taylor rule for the interest rate, as $r_t = \Phi\pi_t - \rho$, with $\rho = \log\beta$. As in the benchmark model, $\Phi > 1$ fixes the amplitude of the response of the Central Bank to inflation π [78]. The linearisation of the Euler equation reads:

$$\pi_t + \frac{1}{\Phi}\kappa_> (\delta_t - \delta_{t-1}) = \frac{1}{\Phi}\mathbb{E}_t [\delta_{t+1} - \delta_t] + \frac{1}{\Phi}\mathbb{E}_t [\pi_{t+1}] \ . \qquad (87)$$

From Eq.(75) one sets $k_> := 3G'(c_>) \geqslant 0$. The inflation is obtained by solving forward in time Eq.(87), which leads to:

$$\pi_t + \frac{\kappa_>}{\Phi}(\delta_t - \delta_{t-1}) = \left(1 - \frac{\kappa_>}{\Phi}\right)\sum_{k=0}^{\infty}\Phi^{-k-1}\mathbb{E}_t[\delta_{t+k+1} - \delta_{t+k}], \quad (88)$$

where δ_t is the output gap defined in (79). In the standard DSGE the function $F_t \to 1$, $\forall t$ and its derivative vanishes. One therefore recovers the standard expression, Eq.(38). The self-reflexive feedback adds a term that depends on the past output gap trend, and changes the coefficient in front of the expected future output gap variations. Interestingly, $1 - F'_>/\Phi$ can become negative for some range of parameters. As already mentioned, extending this result to the scenario where crises are frequent, is really hard. However, we would like to sketch the idea of a possible solution. We define p as the probability of an economic recession occurring between time t and time $t+1$, say $p = T^{-1}(c_> \to c_<) \ll 1$. Consequently, with probability $1-p$ it hovers normally around $c_>$, with small fluctuations. We also assume that $\pi_t \ll 1$. Hence, we approximate the right-hand side of the Euler equation (68) as:

$$\text{RHS} \approx \frac{F(c_t)}{c_>}(1 + \Phi\pi_t)\left((1-p)\mathbb{E}_t^>[(1 - \pi_{t+1} - \delta_{t+1})] + p\frac{c_>}{c_<}\right), \quad (89)$$

where $\mathbb{E}_t^>$ is an expectation conditional to remaining near the high output equilibrium. This eventually leads to an extra term in (88) equal to

$$\delta\pi_t = -\frac{p}{\Phi - 1}\,\frac{c_> - c_<}{c_<}. \qquad (90)$$

As expected, anticipation of possible crises decreases inflation; provided $c_< \ll c_>$ this correction can be substantial even when $p \ll 1$.

3.6 CONCLUSION

The setting presented in this chapter corresponds to a proto DSGE model. Including frictions (like Calvo's staggered price adjustment) would lead to a richer model, with, for example, a modified "New Keynesian Phillips Curve" [78]. Starting from the basic DSGE, we have built a first extension of this model by taking into account confidence effects which, as we have seen, if strong enough can generate drastic drops in consumption, or in other words, economic crises. This effect resolves (at least in part) two of the main criticisms that are addressed to DSGE models. First, the concept of general equilibrium is here overcome by the presence of two distinct equilibria, generating high/low output respectively. Although multiple equilibria can be introduced by changing the form of the feedback function G_t (for example by considering a step function), we didn't investigate such scenario as, our goal, remains to show how, by considering few realistic effects, the outputs of the model can radically change. The second point concerns the exogenous origin of standard DSGE's economic crises. The drops in the consumption of this framework are alimented by endogenous mechanisms and self-propelled. Similarly to the Van der Waals equation – by accounting for simple interactions of the water molecules – succeeds in overcoming the perfect gas equation, in this model the addition of simple feedback effects leads to an economy where recessions are a possible state of the economy. One of the main results of this model is that it suggests alternative, behavioural tools for monetary policy, in particular in crisis time. Beyond adjusting interest rates and money supply, policy makers could also use *narratives* to restore trust,[8] parameterised in our model by the threshold c_0. If, for example, the economy lies in the neighbourhood of the C/B^+ phase boundary (see again Fig. 16), a mild decrease of c_0, engineered by the Central Bank, may help to put back the system on an even keel. However, this model can be improved. Some directions are taken into account in the course of this thesis, others are left for future work. The main directions are:

First, the model is mean field and leaves no room for the heterogeneities. This problem is addressed in the next chapter, Ch.4, where we introduce heterogeneous skills levels and a social network.

Second, the inclusion of market breakdown in crises periods, i.e. allowing for $c_t \neq Y_t$: production and consumption will not match as confidence collapses. This is the subject of an ongoing project conducted within the *EconophysiX* research chair.

Third, within this framework, economic crises have resulted from a decrease in households' propensity to consume. However, one could argue that confidence collapse in 2008 initially affected the supply side. This generalisation represents a challenging problem, and it will be addressed in Ch.5, where we study the possibility of having supply driven recessions. I postpone the discussion to Ch 5.

Finally, within this framework we are incapable of calculating, except in specific situations and/or in an approximate way, the inflation.

8 The importance of narratives in economics was recently stressed in [226]

The absence of a general equilibrium makes the linearisation process impossible, and in particular the estimation of the future expectation $\mathbb{E}_t[\cdot]$ is hard. In particular, without linearisation, the argument $f^i_{t+1}\left(c^i_{t+1}\right)^{-1}/(1+\pi_{t+1})$ of the expectation, needs to be treated as a random variable, leading to the impossibility of computing the inflation. This extension is also the subject of forthcoming work, albeit still at an embryonic stage.

Confidence Collapse in a Multi-household self-reflexive DSGE model: Highlights

In order to bridge the gap between DSGE and ABMs and allow interesting non-linear phenomena, such as trust collapse, to occur within DSGE, in this model we replace the representative household by a collection of homogeneous but interacting households. The setting presented in this chapter corresponds therefore to a proto DSGE model. Here, each household forms its own sentiment, i.e. propensity to consume, by observing what the average consumption of its neighbours was during the previous time period. This opens up the possibility that a relatively small decline in the overall production will lead to a collapse in confidence and a sharp drop in the economic activity. If the effect of feedback is small enough, our model will be equivalent, albeit with increased volatility, to the starting DSGE model. On the other hand, when the strength of the feedback increases, we progressively enter a phase in which the economy admits economic crises, during which output and consumption collapse abruptly but remain short-lived. When the role of feedback grows further, technological shocks can induce persistent economic crises. In such regime the presence of such economic crises occurs even when the noise level is very low, in a *small shocks large business cycle* fashion. The self-reflexive feedback mechanism resolves two of the main criticisms that are addressed to DSGE models. First, the concept of general equilibrium is here overcome by the presence of two distinct equilibria, generating high/low output respectively. The second point concerns the exogenous origin of standard DSGE's economic crises. The drops in the consumption of this framework are alimented by endogenous mechanisms and self-propelled. One of the main results of this model is that it suggests alternative, behavioural tools for monetary policy, in particular in crisis time. Beyond adjusting interest rates and money supply, policy makers could also use *narratives* to restore trust, parameterised in our model by the confidence threshold c_0.

4 HETEROGENEITIES IN A DSGE MODEL WITH FEEDBACK

In this chapter, I discuss the natural extension of the model presented in Ch.3 where we found that the setup, although quite minimal, was already extremely rich, leading to a variety of realistic output dynamics. In particular the appearance of crises where consumption drops as a result of an initial exogenous shock, amplified by a collapse of confidence are effects absent from the basic monetary model, as in Sec.2.2. But while we modelled interactions and feedback loops, we did not account for possible income heterogeneities and network effects, that were averaged out by the mean field approximation. These are key elements to be integrated if one wants to have a realistic description of society.

Here, we build upon such ideas and introduce agents that can be assigned different characteristics, such as skill and social environment. At each time step, the consumption level of each household is partially determined by the past realised consumption of its neighbouring agents in a network of social interactions. As we shall see, we find that the phenomenology, in particular the appearance of endogenous demand-driven crises, is now more complex, with much more structured consumption crashes that are either restricted to some stratum of the population, or affect the whole population, depending on the distribution of wages and the structure of the social network.

This extension follows somewhat the logic described in the introduction, see Sec.1.4.1. Making an analogy with water models, if one considers the model of Ch.3 as the Van der Waals extension of the Ideal gas law (role played by the framework presented in Sec.2.2), the natural step to improve its results is to explicitly account for heterogeneities.

In the following part, I present the solution to the model with general couplings. Only later, once different wage levels have been assigned to agents, we fix the network via the coupling matrix J, starting from the definition of feedback, Eq.(74). *A priori* the value of each coupling J_{ij} is free from constraint. To simplify the problem, in the following we assume that the relationships between agents are matched, i.e. that the matrix J is symmetrical, $J_{ij} = J_{ji} \quad i \neq j$, but in subsequent extensions of the model one can easily extend this setup to a directed graph, i.e. $J_{ij} \neq J_{ji}$.

Moreover, among many possibilities we choose not to assign relative weights and to keep the coupling $J_{ij} = 1/K^i$ where $K^i = K$ is the connectivity, fixed and equal for all agents. However, the coupling matrix will be sparse, as $K \ll M$, with M the total number of households considered. This will impose choices on how to connect the agents. Two possibilities are studied depending on their wage level :

1. agents are segregated and therefore connected to neighbours sharing similar wage levels
2. agents are not segregated and interactions are mostly random.

Last, in the conclusions, I discuss the strengths and weaknesses of our approach compared to the HANK formulation. I also try to illustrate some possibilities to extend the model presented here also accounting for some HANK ingredients.

4.1 SKILLS AND WAGE HETEROGENEITIES

This chapter contains some important differences from the previous setup, and therefore I present all its key steps. The different ingredients of the model are summarised as follows.

4.1.1 *The Households*

As in the previous setup, we consider M households $i = 1, \cdots, M$ who maximise the discounted sum of their present and future utility:

$$U_t^i \quad := \quad f_t^i \log c_t^i - \gamma^i (n_t^i)^2 \tag{91}$$

where, as in Eq.(91), c_t^i, n_t^i are the level of consumption and the amount of working hours of the household i at the time t, f_t^i the (possibly time dependent, see below) utility of consumption and γ^i the disutility of work. Utility maximisation is subject to the classic budget constraint:

$$p_t c_t^i + \frac{B_t^i}{1+r_t} \quad = \quad n_t^i w_t^i + b_{t-1}^i + E_t^i \ , \tag{92}$$

where p_t the price level of goods, w_t^i the nominal wage of the agent i, b_t^i the amount of bonds paying 1 at the time $t+1$, the value of which being $(1+r_t)^{-1}$ at time t, where r_t is the interest rate and the term E_t^i represents an external source of income, see Ch.3 for the explanations over the necessity of this term.

The maximisation is performed using Lagrange multipliers with respect to the quantities c_t^i, n_t^i, b_t^i. This gives the household state equation:

$$c_t^i n_t^i \quad = \quad f_t^i \omega_t^i / \gamma^i \ , \quad i = 1, \dots, M, \tag{93}$$

where, again, $\omega_t^i = w_t^i/p_t$ is the real wages. One also obtains the Euler equation governing intertemporal substitution of consumption:

$$(c_t^i)^{-1} \quad = \quad (1+r_t)\beta\mathbb{E}_t\left[\frac{f_{t+1}^i(c_{t+1}^i)^{-1}}{1+\pi_{t+1}}\right], \tag{94}$$

where $\pi_{t+1} = p_{t+1}/p_t - 1$ is the inflation rate. This equation will not be used in the following part of this chapter, as we will not be concerned with inflation at this stage. The reasons have been discussed in the conclusion of Ch.3.

4.1.2 *The representative Firm*

The production sector is made of a representative firm which uses different skills, corresponding to different productivity levels z^i among agents. We posit that the firm level of production Y_t is given by a Cobb-Douglas[1] function with $\sum_i z^i n_t^i$ as the effective number of working hours:

$$Y_t = z_t \frac{M^\alpha}{1-\alpha}\left(\sum_i z^i n_t^i\right)^{1-\alpha}, \tag{95}$$

where z_t is an overall productivity factor, subject to exogenous shocks, and $\alpha = 1/3$. The pre-factor M^α in (95) ensures that both the aggregate consumption and the production are proportional to the size of the population M. The firm's profit $\mathbb{P}_t$ is then given by:

$$\frac{\mathbb{P}_t}{p_t} := \sum_i c_t^i - \sum_i \omega_t^i n_t^i, \tag{96}$$

The firm maximises $\mathbb{P}_t$ with respect to the individual labour supply n_t^i, under the assumption that the market will clear, i.e.

$$Y_t = \sum_i^M c_t^i\,. \tag{97}$$

Such maximisation provides the following relation between real wage ω_t^i and productivity z^i of each agent:

$$\omega_t^i = z_t\,\frac{z^i}{\mathcal{Z}_t^\alpha}, \qquad \mathcal{Z}_t := \frac{1}{M}\sum_j^M z^j n_t^j \tag{98}$$

Using Eqs. (93) and (98), the market clearing condition (97) becomes:

$$\sum_i^M \frac{z^i}{n_t^i}\left[\left(n_t^i\right)^2 - \frac{f_t^i(1-\alpha)}{\gamma^i}\right] = 0\,. \tag{99}$$

1 One might also consider implementing a constant elasticity of substitution (CES) production function, say $Y_t^\rho = z_t \frac{M^\alpha}{1-\alpha}\left(\sum_i (1-\alpha)(z^i n_t^i)^\rho\right)^{1/\rho}$, $\rho < 0$. This extension/modification is actually quite relevant when we consider the capital dynamics and/or when the skill levels z^i are not fixed in time – see Ch.5.

Given the set of $\{\gamma^i\}$ and $\{f_t^i\}$, Eq. (99) describes a $M-1$ dimensional manifold where the solutions n_t^i must lie.

Now, plugging Eq. (98) into the profit function (96), one finds that

$$\frac{\mathbb{P}_t}{M p_t} = \frac{z_t \alpha}{1-\alpha} \mathcal{Z}_t^{1-\alpha}.$$

Thus, among the set of possible solutions described by (99) we retain the one maximising the sum $\mathcal{Z}_t$. Introducing again Lagrange multipliers, one can show that the optimal solution is given by:

$$n_t^i = F_t^i := \sqrt{(1-\alpha)\frac{f_t^i}{\gamma^i}}, \qquad \forall i, \tag{100}$$

i.e. each term of the sum in Eq. (99) is zero. One might remark that Eq.(100) is coherent with the solution provided in the previous chapter, see Eq.(72) (setting $\alpha = 1/3$).

Combining Eqs. (98) and (93) finally yields:

$$c_t^i = z_t \frac{z^i F_t^i}{1-\alpha} \left(\frac{1}{M} \sum_j^M z^j F_t^j \right)^{-\alpha}. \tag{101}$$

Eqs. (98) and (101) are our central theoretical results. Eq. (101), which appears to be new, gives the general solution of a generalised DSGE model with many heterogeneous agents, while keeping most of the original DSGE fully rational agent paradigm intact up to now.

To move forward, we need to specify the distribution of skills z^i over the population, as well as the dynamics of the overall productivity factor z_t.

Since in our model real wages are proportional to skills (see Eq. (98)) we take inspiration from empirical data, which shows that wages follow an exponential distribution, except in the extreme tails where it becomes fatter (possibly Pareto-like), in part due to returns on investment, see e.g. [227]. To keep the model as parsimonious as possible, we therefore assume that the distribution of z^i in the population is given by:

$$\rho\left(z^i\right) = \begin{cases} \frac{1}{\mu} \exp\left(\frac{z_{\min}-z^i}{\mu}\right) & z^i \geqslant z_{\min} \\ 0 & z^i < z_{\min}. \end{cases} \tag{102}$$

This exponential distribution has a mean given by $\mathbb{E}[z] = z_{\min} + \mu$, which can be considered as a proxy for the GDP per household of the corresponding economy. The distribution of wages is also characterised by a Gini coefficient $\mathcal{G}$, which is a measure of the inequalities in our economy. A schematic representation of the mathematical meaning of the Gini coefficient is shown in Fig.18. In particular,

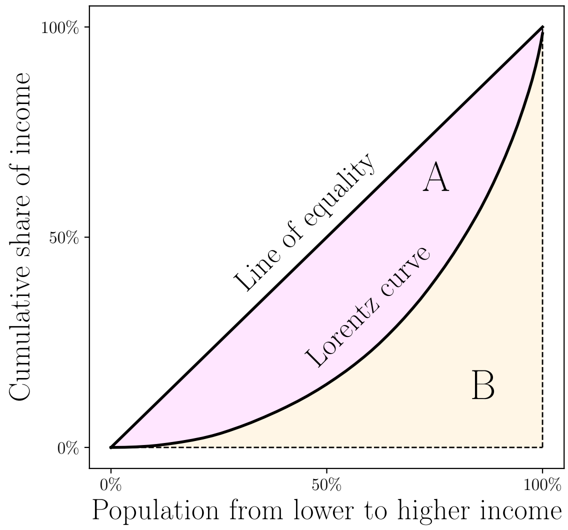

Figure 18: The figure shows the derivation of the Gini index. On the horizontal axis, the population, ordered by increasing z^i. On the vertical axis the cumulative share of income, given by the Lorentz curve, whose components $\{L^i\}$ are computed as $L^i := \sum_{j=0}^{i} z^i / \sum_{j=0}^{M} z^j$. If $z^i = z$, $\forall i$ then the increments are constant and the Lorentz curve is a straight line of angular coefficient 1 (as shown on the figure). On the other limit, when the totality of the wealth is accumulated by one single agent, the corresponding Lorentz curve is given by the dashed black lines. For any other distribution of the inequalities, the corresponding Lorentz curve lies below the "line of equality". The Gini coefficient is a measure of such inequalities, and it is computed as the ratio $A/(A+B)$, where A and B are the areas coloured in the figure.

when the inequalities are exponentially distributed, the Gini coefficient takes a simple form, and it reads:

$$\mathcal{G} = \frac{\mu}{2\mathbb{E}[z]} = \frac{1}{2}\frac{\mu}{\mu + z_{\min}}. \tag{103}$$

Hence, $\mathcal{G} \to 0$ when $\mu \ll z_{\min}$ (egalitarian society) and $\mathcal{G} \to 50\%$ when $\mu \gg z_{\min}$. Stronger inequalities (i.e. $50\% < \mathcal{G} \leqslant 1$) would require a different functional form, with, for example, extra power-law tails, or a Dirac mass at $z = z_{\min}$. The quasi-totality of European countries have a Gini index ranging between 24% and 35% while more unequal societies, such as the US, have Gini's $> 40\%$ [228].

Exogenous shocks are encoded into the idiosyncratic noise z_t that we write as $z_t = e^{\xi_t}$, where ξ_t follows an AR(1) process:

$$\xi_t = \eta_z \xi_{t-1} + \sqrt{1-\eta_z^2}\,\mathcal{N}\left(0, \sigma_z^2\right)\ , \tag{104}$$

where we fix $\eta_z = 0.2$[2] (the parameter η_z only affect the timescale of the memory kernel of the stochastic process). This corresponds to

2 As a remark: the value of η_z is smaller if confronted to Ch.3. This choice is justified by the increasing complexity of the numerical simulations performed here. Hetero-

assuming that all individual productiveness z^i are subject to the same exogenous shock. One extension of this model is to consider a setup where the different skills are affected by different shocks.

Note that the most probable value of z_t is unity (i.e. $\xi_t = 0$), which corresponds to what we will call "normal" or "baseline" conditions.

4.2 SOCIAL NETWORK AND SELF-REFLEXIVITY

We now discuss the specific form of the consumption propensity f_t^i or, equivalently, its (re-scaled) square-root F_t^i defined in Eq. (100). Following Ch. 3, we assume that the consumption propensity of agent i at time t depends on the realised consumption at time $t-1$ of some other agents of the economy (self-reflexivity), which are coupled to i via an interaction network J_{ij}:

$$F_t^i \equiv \mathcal{F}^i\left(\frac{1}{K_i}\sum_{j(\neq i)} J_{ij} c_{t-1}^j\right), \quad K_i := \sum_{j(\neq i)} J_{ij}, \tag{105}$$

where $\mathcal{F}$ is a certain function the argument of which is the local average of the consumption at time $t-1$ of "neighbours" on the network, i.e. the agents j for which J_{ij} is different from zero. The specific choice of this interaction network will be discussed in details below. Here we start by focusing on the properties of the feedback function $\mathcal{F}^i$. In the previous chapter we have shown that a generic S-shaped function suffices to induce multiple equilibria, with stochastic switches (corresponding to economic crises and recoveries) between them. As in Ch.3, we choose a logistic function of the form:

$$\mathcal{F}^i(x) = \frac{1}{2}\left[(\nu_{\max}^i - \nu_{\min}^i)\tanh\left(\theta^i(x - c_0^i)\right) + (\nu_{\min}^i + \nu_{\max}^i)\right]. \tag{106}$$

The parameters $\nu_{\min}^i > 0$ and $\nu_{\max}^i > \nu_{\min}^i$ represent the minimum and maximum levels of labour that household i can possibly provide, see Eq. (100); c_0^i is the individual confidence threshold where the concavity of $\mathcal{F}(c)$ changes. Similarly to the previous setup, $c^i > c_0^i$ tends to induce a high confidence state, while $c^i < c_0^i$ a low confidence state. $\theta^i > 0$ is the steepness of the function $\mathcal{F}^i$ close to the threshold level, setting the width over which the transition from low confidence to high confidence takes place, and is related to the agents' sensitivity to consumption's changes. In order to fully specify the model, we still need to define the interaction network i.e. the link variables J_{ij}. We base our choice on a number of studies indicating that households sharing the same level of wealth tend to cluster together (see for example [229, 230]). For example, in large cities, the real estate market is such that people sharing a comparable level of income populate the

geneities have a major impact in the running time of the simulations and reducing η_z allows collecting statistic faster, reducing by some factors the numerical time.

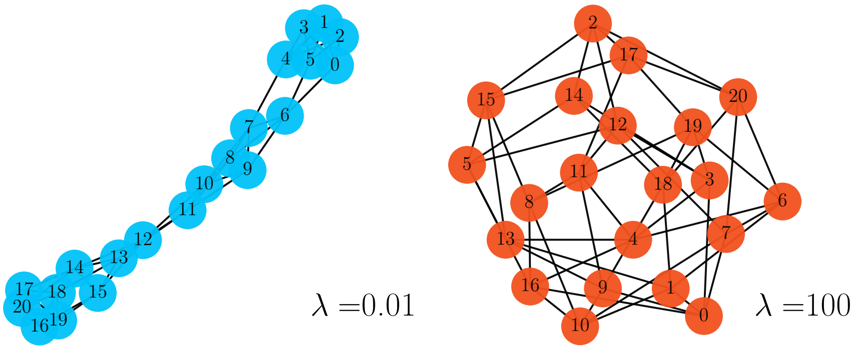

Figure 19: The two graphs show two sketched of graphs obtained for different levels of segregation. Specifically $M = 20$, $K = 5$. Each node corresponds to the same distribution of $\{z^i\}$ and they are sorted in increasing order, i.e. $z^0 < z^1 < \cdots < z^{19} < z^{20}$. The left panel shows the segregated network corresponding to $\lambda = 0.01$, while the right one displays the non-segregated case, $\lambda = 100$.

same neighbourhoods, and therefore attend the same schools, facilities and many other public infrastructures. Following this reasoning, we set $J_{ij} = 1$ with probability p_{ij} and $J_{ij} = 0$ with probability $1 - p_{ij}$, where p_{ij} is given by

$$p_{ij} \propto \frac{K}{M} \exp\left(-\frac{|z^i - z^j|}{\lambda(z^i + z^j)}\right) . \tag{107}$$

This implies that agents with similar wages (i.e. $|z^i - z^j|/(z^i + z^j)$ small) are more likely to be in contact (i.e. $J_{ij} = 1$) than agents in different social classes (i.e. $|z^i - z^j|/(z^i + z^j)$ large). The stratification and the level of segregation of the society is tuned by the parameter λ: when $\lambda \gg 1$, wage differences become irrelevant, whereas when $\lambda \ll 1$ interactions are almost exclusively within the same social group. Fig.19 graphically displays the differences between a segregated and a non-segregated social network. The factor K/M in Eq. (107) ensures that each household interacts with a small average number K of other households. In fact, to be more precise, in the following we will consider "random-regular graphs" of fixed connectivity K [231], which are defined as a graph chosen uniformly at random among all possible graphs of M nodes such that each node has *exactly* K edges connecting it to its neighbours.

The procedure that we implement to build the network goes as follows:

1. We first assign a wage level z^i to each of the M nodes of the network, which are i.i.d. variables extracted from the distribution (102).

2. We build a random-regular graph of fixed connectivity K.

3. The links are then rewired through a Monte Carlo algorithm. In order to keep the connectivity fixed, we proceed as follows: We assign to any configuration an energy equal to $H = \sum_{\langle i,j \rangle} |z^i - z^j|/(z^i + z^j)$, where $\langle i, j \rangle$ designates pairs of neighbouring agents. From the randomly generated graph, we pick at random two links, say $i \to j$ and $k \to \ell$ and we swap the connections to $i \to k$ and $j \to \ell$. We compare the energies of the old configuration, H^{old} with that of the rewired configuration, H^{new}. The new configuration is kept with probability $\min(1, e^{(H^{\text{old}} - H^{\text{new}})/\lambda})$. This process is repeated until a stationary state is reached. It is possible to show rigorously that the probability distribution at equilibrium is given by Eq. (107). Results for $\lambda = 0.01$ and $\lambda = 100$ are displayed on Fig.19 [3].

While one could have made a number of alternative choices to model heterogeneities in both income and social interactions, we believe the setting introduced above is general and simple enough, and contains the essential features that we want to account for.

It is worth adding that here we only focus on *symmetric* interactions matrices (i.e. if j influences i then i equally influences j). Another possible extension of the current setup is to consider the generalisation to directed networks.

4.3 PARAMETER SPECIFICATIONS

In this section, we propose reasonable and parsimonious specifications for the different parameters defined in the previous section.

4.3.1 *Wage distribution*

The exponential distribution of wages, Eq.(102) has two parameters, governing the average wage and the Gini coefficient. To disentangle the two effects, we first investigate the model with a fixed value of $\mathbb{E}[z]$, arbitrarily set to 2, and vary μ in the interval $[0.2, 1.8]$, corresponding to Gini coefficients (given in this case by $\mathcal{G} = \mu/4$) between 5% and 45%. As discussed above, the average productivity level $\mathbb{E}[z]$ represents the average income and is essentially proportional to the GDP per capita of one country. When comparing the predictions of our model to real-world data, we will relax the constant salary mass hypothesis and impose it to be proportional to the GDP/capita. This extension is discussed in detail in the last part of the chapter.

3 Alternatively instead of rewiring the links (computationally expensive), one can swap the positions of agents via the same procedure. This allows a computational gain with consequent time saving.

4.3.2 *Feedback function*

The feedback function is specified, for each agent i, by four parameters: ν^i_{max}, ν^i_{min}, c^i_0 and θ^i. We assume that the minimum amount of labour provided by an agent i, ν^i_{min}, is the same for each agent and equal to zero. (For practical convenience, we fix it to a very small value, $\nu^i_{min} = 10^{-3}$.) Similarly, the maximum amount of labour ν^i_{max} can be set to 1, independently of i. Using Eq. (128), this implies that consumption c^i in booming times is proportional to income z^i, as expected.

The most important parameters of the feedback functions are c^i_0 and θ^i. We assume that these parameters only depend on the income of each agent, as detailed below. In this way we are able to reduce drastically the number of free parameters of the model.

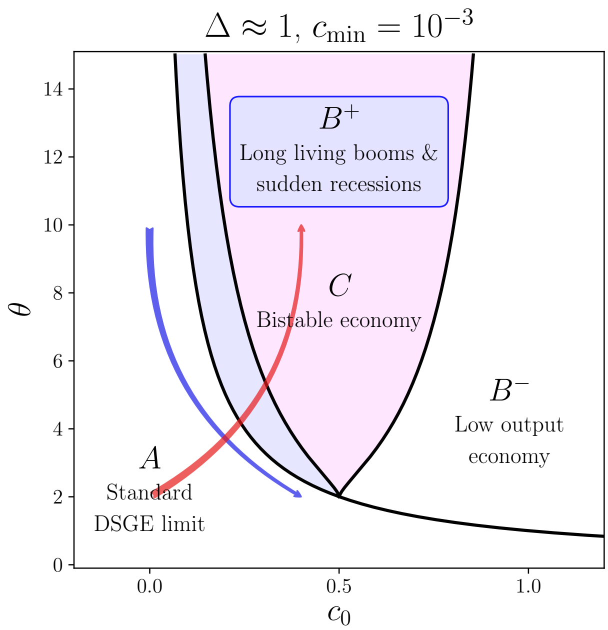

Figure 20: In the figure, we show a sketch of the phase diagram as in the homogeneous case, with highlighted the different phases and their properties. In red and blue, we draw two possibilities for the choice of the $\theta(c_0)$. The arrows point in the same direction as the increase in wages' levels. This figure is meant to be a guide to help the reader to understand the model and the choice of parameters.

The c^i_0's play a key role as they correspond to the threshold below which the households' confidence collapses. In a recent article by D. Jacobe,[4] it is reported that in the early stages of the 2008 GFC, the wealthier part of the population was also the most pessimistic about the state of the economy. To account for this effect, we set the confi-

4 `https://news.gallup.com/poll/111895/HigherIncome-Americans-Turning-More-Pessimistic.aspx`

dence threshold of each household c_0^i to be an increasing function of its income level, and hence of its productivity level z^i, modulated by an exponent $\beta_1 \geqslant 0$:

$$c_0^i = \bar{c}_0 (z^i)^{\beta_1} , \tag{108}$$

where $\bar{c}_0$ is a global trust level that we assume to be determined by country specific economic policies, culture, etc. The larger the value of β_1, the stronger the dependence of the confidence threshold on income.

In order to gain some intuition about the specification of the sensitivity parameters θ^i, we use as a guide the phase diagram established in the homogeneous case in Ch.3, recalled in Fig. 20. Depending on the values of θ and c_0, one can distinguish four zones in the phase diagram that encode different properties of the economy. Those regions correspond, by construction, to the phases I described in the previous chapter. Here I limit to recall their properties: (i) the phase A delimits the area of the standard DSGE model, where we do not observe any economic crisis, (ii) the zone B^+ allows for short-lived economical recessions, (iii) the C phase admits a second equilibrium and correspondingly allows for crisis and economical recoveries with comparable probability and duration. Finally, (iv) zone B^- represents the set of parameters for which the system is systematically in a state of crisis. Over such a phase diagram, we draw two possible "trajectories" for $\theta(c_0)$: in blue a convex decreasing relation and in red an increasing one. Both curves cross different phase transition lines, but the blue one seems a more natural choice. [5] Actually, we find that along the red curve it is almost impossible to find a set of parameters for which all the agents belong to the same phase and, moreover, the richest part of the population is systematically exposed to the economic crisis regardless of the choice of parameters.

For these reasons, we discard the "red" option and parameterise $\theta^i(c_0^i)$ as:

$$\theta^i(c_0^i) = \bar{\theta}(z^i)^{-\beta_2} = \bar{\theta}\left(\frac{c_0^i}{\bar{c}_0}\right)^{-\frac{\beta_2}{\beta_1}} , \tag{109}$$

where $\bar{\theta}$ represents the global sensitivity scale and the exponent $\beta_2 > 0$ enforces a monotonic decreasing dependence between θ^i and incomes. When $\beta_2 = \beta_1$, the width θ^{-1} of the transition region scales as the consumption threshold c_0 itself. When $\beta_2 < \beta_1$ on the other had (as we will find empirically), this width increases slower that c_0, meaning that high incomes are (on a relative basis) more sensitive than low incomes to a drop of consumption of their neighbours.

5 Here we only considered two possible patterns of c_0^i and θ^i. However, this model, being extremely versatile, can be easily implemented with different choices over the definitions of c_0^i and θ^i.

Visually, when $\bar{\theta}$ increases the blue line is globally shifted upwards, while if $\bar{c}_0$ increases it is shifted to the right. When $\beta_1, \beta_2 \to 0$, $c_0^i = \bar{c}_0$ and $\theta^i = \bar{\theta}$, behavioural heterogeneities are switched off and the model leads to a phenomenology very similar to the one reported in Ch.3. We are thus left at this stage with only four parameters: $\beta_1, \beta_2, \bar{c}_0, \bar{\theta}$. Although seemingly restrictive, this setting gives rise to a rich phenomenology that we are going to analyse in the next sections.

4.4 CHARACTERISING CRISES TYPOLOGIES

4.4.1 *Numerical results*

In the previous chapter, we showed how the introduction of the feedback function can destabilise the standard DSGE equilibrium. In the C phase, the self-consistent solution for the consumption has two fixed points, allowing the system to switch from a high to a low consumption state. As shown in the phase diagram of Fig. 20, the confidence threshold c_0 modulates the probability of jumping from a high consumption state to a recession regime (this probability increases with c_0). Here this mechanism remains unchanged, but the chain of events that bring the consumption of agent i to collapse is more intricate. Figure 21 gives some insights about the possible scenarios.

The three pairs of panels shown in Fig. 21 display the crisis propagation for three different choices of the parameters β_1 and β_2, for fixed values of $\bar{\theta}$ and $\bar{c}_0$. We also compare segregated (left column) and non-segregated networks (right column). In the segregated case, we observe that crises form suddenly and then slowly abate.

Changing the values of β_1 and β_2, e.g. moving from the upper to the lower panels, affects dramatically which social class undergoes consumption-driven crisis. In the upper two panels, for instance, crises spread almost exclusively from the poorest end of the population towards the middle class and only sporadically affect the whole system. In the central panels, we observe a different scenario: the recessions originate with almost the same frequency from the richest or the poorest part of the population and affect both social classes with the same intensity. Finally, in the lowest panels, crises always start from the richest agents and propagate towards the middle-class and only in very few cases affect the whole population.

In the right column of Fig. 21 (non-segregated networks) we observe that for the three choices of β_1 and β_2, only the shape and duration of the recessions are affected compared to the segregated case. Recessions spread more uniformly and are shorter.

Below, we explain how to rationalise these observations.

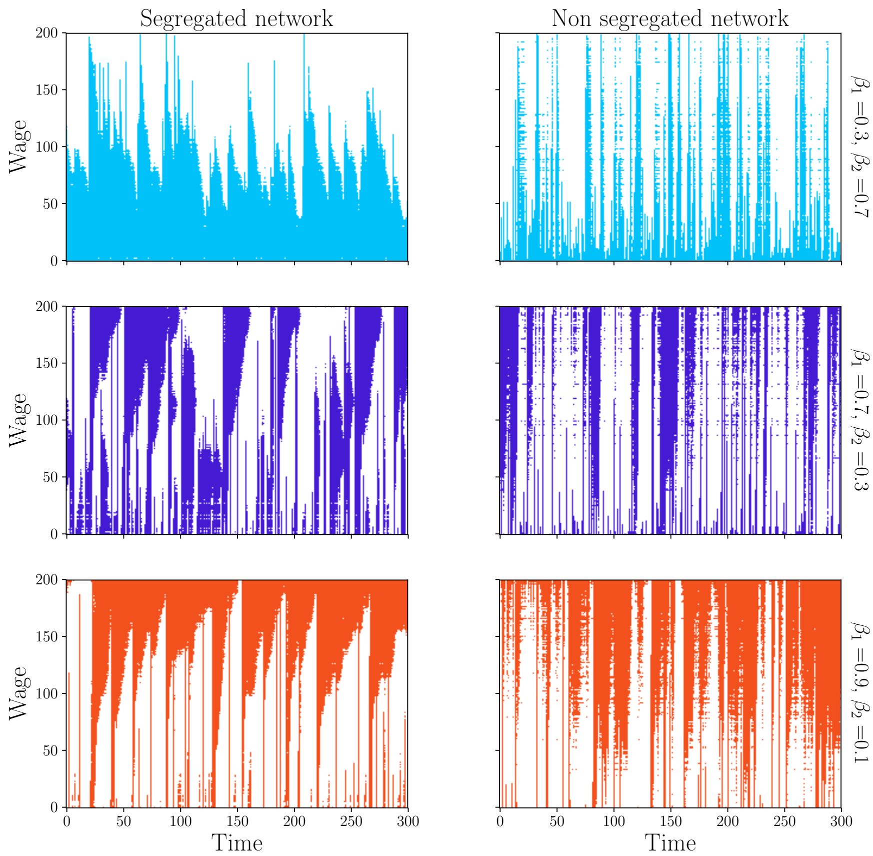

Figure 21: The graphs show the crises dynamics for three choices of the parameters, together with the relative phase diagram, using the same colour code. In the main panels, the abscissa is time and the ordinate are households, sorted by increasing wage. Colour appears when one agent's consumption drops below its corresponding threshold. The three dynamics differ only by the choice of the couple β_1 and β_2, while the global levels of $\bar{\theta} = 4$ and $\bar{c}_0 = 0.5$ are kept constant, together with the level of income inequalities $\mu = 1.5$. For the leftmost set of graphs, the network is segregated, $\lambda = 0.01$, while the right ones are with $\lambda = 100$. Note that the typology of crises changes substantially between the two cases.

4.4.2 *A path across the phase diagram*

In the limit of strongly segregated networks ($\lambda \ll 1$), the only connections are between agents that have very similar income and, therefore, very similar values of θ and c_0. Their consumption obey a self-consistent equation very similar to the one presented in the previous Ch.3, but with wage dependent parameters:

$$c(z) = \gamma z \mathcal{F}\left(c(z)|\theta(z), c_0(z)\right), \qquad \gamma := \frac{\mathbb{E}[zF]^{-\alpha}}{1-\alpha}. \tag{110}$$

where the time dependence of the consumption c is neglected in the absence of productivity shocks. Depending on the choice of the pa-

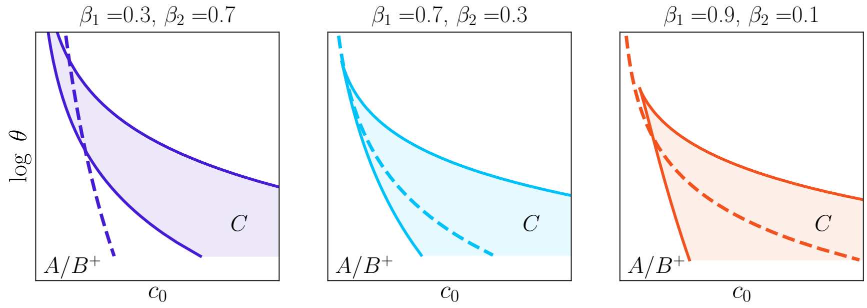

Figure 22: The three panels sketch the phase diagram. The plain lines indicate the boundaries of the C phase, while the dashed line represents the locus of c_0 and θ for different households. The colour code matches the one used to illustrate the dynamics in Fig.21.

rameters θ and c_0, Eq. (110) can have 1, 2 or 3 solutions. In the homogeneous case, the representative agent occupies a single point in the phase diagram of Fig. 20. In the limit of strong segregation, each social class occupies a different spot of the phase diagram. The union of these spots form the lines drawn in Fig. 20.

The shape and the location of these lines strongly depends on the values of β_1 and β_2, as shown on the rightmost panels of Fig. 22 (the colour of each of those panels are chosen to match the one of the corresponding dynamics). The left graph shows that agents with lower income are living in the C phase (bi-stable economy) while the steep decrease of θ with z, allows the richer households to cross the $C \to B^+$ phase line. This reflects the dynamics shown on the corresponding panels.

In the middle panel, the whole population lies within the C phase, explaining why crises form from both sides of the income spectrum. Finally, the right panel, although very similar to the previous case, reveals an important difference: households with a lower salary are closer to the line separating the phases C and A/B^+. This affects the probability for the lower income class to suffer a drop of consumption, which is decreased compared to the middle panel case.

4.4.3 *The myopic effect of segregation*

The arguments given above are rigorous in the limit of segregated societies, but cannot explain the strong influence of λ on the typology of crises. As revealed by Fig. 21, changing the structure of the interactions leads to a drastic modification of the shape and duration of the recession spikes. In fact, by varying the segregation of the network, we affect the correlation between the average income of the households (on which agents' trust is based) and one's own income.

In a clustered society ($\lambda \ll 1$), the aggregate consumption of a family's neighbours is similar to the consumption of the agent itself. This creates an effect of myopia, as agents probe the health of the economy only to a local scale. In this case, contagion effects are maximised. As social segregation increases, the fragility of the social class most exposed to an economic recession, as each agent is connected to others sharing a comparable wage and living in the same phase. Hence, we expect a sort of avalanche effect, as one agent's drop in consumption induces, with higher probability, the trust collapse of its neighbours.

On the contrary, in a non-segregated society, agents base their trust in the economy by picking a few agents chosen at random. This allows, for instance, the consumption of a low-wage person to be boosted by that of a wealthier neighbour, improving his own trust in the economy, and *vice versa*. Diversification improves stability in this case: the domino effect is much weaker in the non-segregated network due to the fact that heterogeneous income level of the neighbours decreases the effects of the feedback function. Therefore, the crises in the non-segregated case are shorter and rarer, and can only be produced by a stronger exogenous shock.

The main message is thus that diversification of information sources increases resilience. In fact, comparing the upper and the bottom panels of Fig. 21 we see that several small spikes in a non-segregated society coalesce in a unique recession event when segregation is strong, due to the avalanche effect described above.

4.4.4 *Exogenous shocks and global crises*

After having investigated which households are the most affected by an economic recession, we now discuss how the size of the crises depends on those parameters, regardless of the social class. In order to do that, we introduce the quantity $x_{<,t}$, defined as the fraction of households being in a low consumption state at time t, independently of the income level:

$$x_{<,t} := \frac{1}{M} \sum_{i=0}^{M} \Theta(c_0^i - c_t^i) \,, \tag{111}$$

where Θ is the Heaviside function: $\Theta(x > 0) = 1$ and $\Theta(x \leqslant 0) = 0$. In the panels of Fig. 23 we draw the (logarithm of the) probability $p(x_<)$ of observing a crisis of "size" $x_<$, for different values of the income inequalities, of the exponents β_1 and β_2, and for segregated and non-segregated networks.

In the panels where $\beta_1 = 0.1$ (first two rows panels) we observe a transition from a uni-modal to a bi-modal distribution as μ is decreased, i.e. as inequalities decrease. For low values of μ the probability distribution has two peaks: the first one in $x_< \approx 0$, describing a well-functioning economy where most of the agents are in the high-consumption state, and the second one in $x_< \approx 1$, corresponding to global crisis where nearly all agents are in a recession state. In

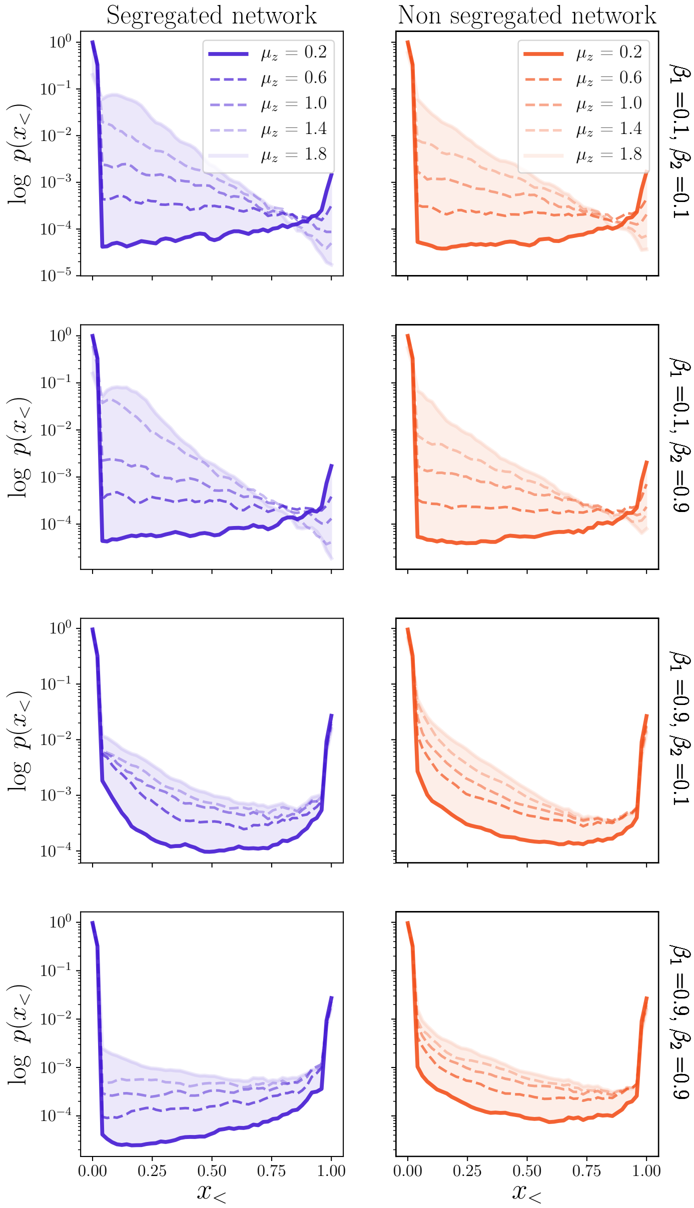

Figure 23: The panels show the probability distribution of the crisis size, $x_<$, for different choices of the parameters. The blue curves show the results for the segregated case, $\lambda = 0.01$, while the red ones represent the non-segregated scenario, $\lambda = 100$. Each panel is dedicated to a couple β_1, β_2 for five values of μ, ranging from 0.2 to 1.8. $\bar{\theta} = 4$ and $\bar{c}_0$ are kept fixed.

the uni-modal regime at larger μ, instead, the probability becomes roughly exponential in the crisis size $x_<$.

In the uni-modal regime most crises only affect a limited fraction of the population, and only very rarely hit the whole population (as in the examples shown in Fig. 21). Conversely, in the bi-modal regime, recessions are mostly global. This can be rationalised by recalling that in the limit $\mu \to 0$ all the agents have the same income, skills, and baseline consumption levels. We thus recover the results of the homogeneous model [1] in which only two states are possible (the whole population is in the good state or in the low-consumption one) and $x_<$ is either 0 or 1.

Comparing the results here obtained with the ones presented in Ch.3, we remark that introducing wage inequalities allows for the possibility of having intermediate crises, that only affect a finite portion of the agents, thereby reducing the probability of a global crunch.

Comparing the right and left panels, we notice that the level of segregation does not have a major influence on the shape of the distributions $p(x_<)$ (even though the crisis dynamics itself is strongly affected by λ, as shown in Fig. 21).

At this point, the question that we still need to address is: what parameters affect, and how, the probability of having a global consumption crisis?

To answer this question, we introduce the probability $\mathcal{P}$ of observing a global crisis, which is defined as an event in which the consumption of more than 80% of the population drops below their level c_0^i, i.e.

$$\mathcal{P} := \int_{0.8}^{1} p(x_<)\, dx_<. \tag{112}$$

$\mathcal{P}$ plays the role of an order parameter for the uni-modal/bi-modal transition described above, as it is small in the uni-modal regime and takes appreciable values for bi-modal distributions.

Similarly to the homogeneous case, the crisis probability strongly depends on the amplitude of the external shocks σ_z and on the global confidence threshold $\bar{c}_0$. In the lower panels of Fig. 23 we plot the dependence of $\mathcal{P}$ on σ_z for different choices of the other parameters. For the sake of clarity, in each panel we keep three of the four parameters $\bar{c}_0$, $\bar{\theta}$, β_1 and β_2 fixed, and let one of them vary (as indicated in the legends). In each panel, we also show different curves corresponding to several values of μ.

We find that the probability of having a global crisis becomes nonzero beyond a certain critical amplitude of the noise, σ_z^c. We further observe that σ_z^c decreases with increasing $\bar{c}_0$ and/or $\bar{\theta}$. This result agrees with the simple intuition that for lower global confidence, or, similarly, stronger global sensitivity, global crises can be triggered by a smaller exogenous shock. In other words, referring again to the phase diagram shown in Fig. 22, an increase in $\bar{c}_0$ at constant $\bar{\theta}$ shifts the system to the right, whereas an increase of $\bar{\theta}$ shifts the system upwards. Households are thus pushed deeper into the C phase and are more frequently exposed to global economic crises. The effect of β_1 and β_2 on $\mathcal{P}$ and σ_z^c is rather weak, as is the influence of segregation – see Fig. 24. Hence, β_1 and β_2 have an impact on the social class that is more frequently affected by the crises, but not on the probability of having a global recession.

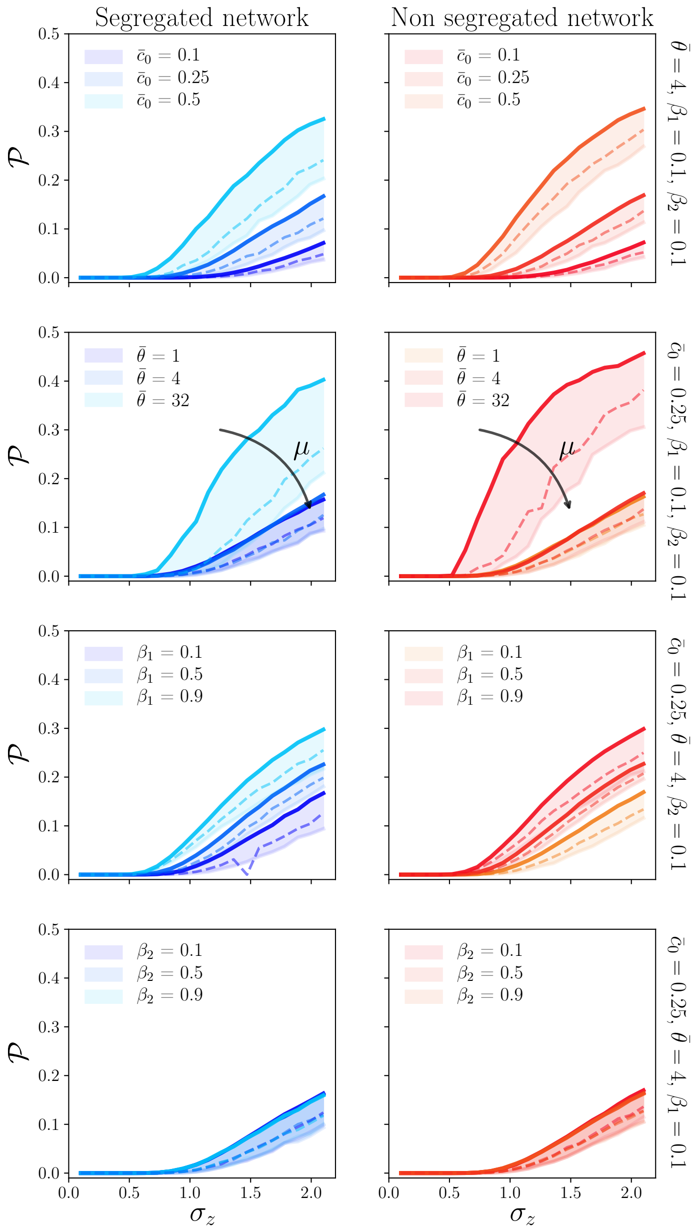

Figure 24: The panels show the probability of a global crisis $\mathcal{P}$ as a function of the exogenous shock amplitude $\sigma \in [0.1, 2.1]$. We restrict the analysis to three values of μ: 0.2, 1.0 and 1.8. In each panel the intensity of the colour reflects the income inequality μ as in Fig. 23. In each graph the global colour represents the level of segregation: when blue $\lambda = 0.01$, when red $\lambda = 100$. We study the dependence of $\mathcal{P}$ on four parameters: in each couple of graphs (segregated and non-segregated network) we let one parameter between $\bar{c}_0$, β_1, $\bar{\theta}$ and β_2 vary, and we keep the other three constants. When kept constants, the parameters take the following values: $\bar{c}_0 = 0.25$, $\bar{\theta} = 4$ and $\beta_1 = \beta_2 = 0.1$. The varying parameter is represented with different line style, and the legend is shown within each panel.

Finally, the same figure shows that increasing inequalities (higher values of μ) generally lowers the probability $\mathcal{P}$ of having a global crisis, and increases the critical value σ_z^c. This is consistent with the content of the upper panels: as discussed above, increasing μ favours (in our model) the formations of smaller crises, that only affect a certain fraction of the population and reduces the exposure to global crises of the whole population.

However, the conclusion that more inequalities lead to a smaller probability of global crises is possibly misleading, as it neglects an important effect not accounted for in our model, namely the dependence of the global "panic" level $\bar{c}_0$ on the Gini coefficient $\mathcal{G}$. Indeed, a recent report from the OECD pointed out that:

"[...] societies with a strong middle class also experience higher levels of social trust [...]. Today, however, middle-class households became increasingly anxious about their economic situation [...] given that middle incomes have not benefited from economic growth as much as upper incomes [...]" [232].

In other words, higher income inequality should also raise the value of confidence threshold $\bar{c}_0$, leading to a more unstable society. It is not obvious which one of the two effects (stabilizing vs. destabilizing) is dominating. We in fact suspect that the influence of inequalities on $\bar{c}_0$ is non-linear, and only mild when inequalities are moderate.

4.5 EMPIRICAL DATA

The results of the previous sections show that the model can reproduce a broad spectrum of possible scenarios for the formation and the propagation of economic crises across a society with stratified income levels.

In this final section we will exploit such versatility to compare the output of the model with real data, discussing differences and similarities when key parameters are modified. This exercise is not easy, as empirical data on the level of consumption for different income groups is not always available and/or complete for each country. On the other hand, data on income distribution exists. Our aim here will be to exploit the available data to show that there is a region of the parameter space that is consistent with empirical observations on the relative drop of consumption of the poorest compared to that of the richest during a crisis.

However, since income data also includes returns from financial investments, our assumption that income has an exponential distribution is not adapted to describe the high tail, for which a power-law is more adequate [227]. It may, in fact, be that a substantial part of the effect reported below results from financial losses, and not from the contagion effect captured by our model – except perhaps in an effective way, see below.

The data set we have explored is available from the website *Our World in Data* [233]. It provides information regarding the consumption of the richest and the poorest decile, called respectively $c^a_{90,t}$ and $c^a_{10,t}$, for a large range of years t and countries a. Furthermore, we refer to levels of GDP per capita, which is available on the same platform. The main interest of our study is to understand how heterogeneities and income inequalities affect the response of the population in a crisis scenario.

The quantities $c^a_{90,t}$ and $c^a_{10,t}$ are typically provided for each year t, but in cases where they are omitted, we interpolate the missing data point of the two closest available data points.[6] In order to track and compare the time evolution of the consumption of the highest and the lowest deciles, we compute, for each year t and country a, the relative difference:

$$\delta c^a_{\star,t} := \frac{c^a_{\star,t} - c^a_{\star,t-1}}{c^a_{\star,t-1}}, \quad \star = 10, 90 . \tag{113}$$

It is clear from the definition that when $\delta c^a_{\star,t}$ assumes negative values it means that the consumption of the $\star$-th decile of the country a has dropped in the time-lapse of one year. We define such an event as a recession that affected at least one extreme of the population, i.e. either $\delta c^a_{90,t} < 0$ or $\delta c^a_{10,t} < 0$. To monitor how unequally such crises affect the population, we introduce the indicator Δ^a_t defined as:

$$\Delta^a_t = \delta c^a_{90,t} - \delta c^a_{10,t} . \tag{114}$$

This quantity Δ^a_t captures how economic crises spread in the society:

1. If $\Delta^a_t < 0$ the richest decile undergoes a greater relative drop in consumption during the crisis compared to the poorest decile.

2. If $\Delta^a_t > 0$ the poorest decile experiences the largest relative consumption drop.

The other key elements of our model are the segregation index λ (for which we have no direct data) and income inequalities, described by the Gini index $\mathcal{G}^a_t$ associated to the country a at date t. We also cut our sample into rich countries, with GDP/cap. larger than the median, and poor countries, with GDP/cap. less than the median.

6 We use the logarithm for interpolation because we want to keep track of the exponential growth of consumption. For example, if the natural progression is 2, $\star$, 8, where $\star$ represents the missing information, using this method we find $\star = 4$, which seems more reasonable. Without interpolating the data, the number of points with complete information for GDP/capita, $\mathcal{G}$ and Δ is 113. However, if we interpolate the missing information, this number rises to 206. 10 of these countries have $\mathcal{G}$ greater than 50% and are therefore not exploited, as our exponential model does not account for Gini's larger than 50%.

Segregated Network

GDP/capita <median value

$\beta_1 \backslash \beta_2$	0.1	0.3	0.5	0.7	0.9
0.1	-0.0004	0.0018	0.0135	-0.0032	-0.0425
0.3	**-0.0017**	**-0.0011**	0.0039	0.016	0.0367
0.5	-0.0006	**-0.0021**	**-0.0008**	0.0192	0.0394
0.7	0.0007	0.0004	-0.0003	-0.0007	0.0669
0.9	-0.0065	0.0002	0.0015	0.0011	0.0009

GDP/capita $\geqslant$ median value

$\beta_1 \backslash \beta_2$	0.1	0.3	0.5	0.7	0.9
0.1	-0.0007	0.0019	0.0146	0.009	0.0274
0.3	**-0.0022**	**-0.0011**	0.0021	0.0074	0036
0.5	-0.0352	**-0.0021**	**-0.0011**	0.0017	0.0194
0.7	-0.1153	-0.0671	-0.0045	-0.0005	0.0023
0.9	-0.0732	-0.0674	-0.043	-0.005	0.0004

Table 2: This set of tables document the coefficients of linear regressions of numerical Δs as a function of the Gini coefficient $\mathcal{G}$, for different choices of parameters: β_1, β_2 and $\lambda = 0.01$ (segregated network). We fix as constants: $\bar{c}_0 = 0.5$, $\bar{\theta} = 4$, $\sigma = 1$. For each combination of parameters, several independent simulations are performed, during which the time evolution of Δ_t is calculated and then averaged, conditioned to a crisis, i.e. either $\delta c_{90} < 0$ or $\delta c_{10} < 0$. We further distinguish between countries having a GDP/cap. higher and lower than the median value of the available data. The reference value of the regression for empirical data is **-0.0018** if GDP/capita <median value and **-0.0017** otherwise.

The processed data is displayed in Fig. 25 where we show Δ_t^a versus $\mathcal{G}_t^a$, for all available years t and countries a (without distinctions). To better visualise the GDPs we set the size of the markers (x) proportional to its value, and we choose to adapt the grey level accordingly: light grey corresponds to high GDPs, and dark grey to low GDPs. For the following discussion, we will refer to Δ as being the set given by $\Delta_t^a, \forall a, t$.

We observe that Δ exhibits a *negative* overall correlation with $\mathcal{G}$: $\mathbb{C}(\Delta, \mathcal{G}) \approx -0.126$. This means, perhaps unexpectedly, that with the increase of inequalities the relative response to a recession is in favour of the poorest.[7] This is compatible with our assumption that $\beta_1 > 0$, i.e. that the confidence threshold of the high earners is higher than that of the low earners (meaning that transition to a low consumption

7 A double regression against both Gini and GDP/cap. shows that the direct impact of GDP on Δ can be safely neglected.

Non-segregated Network

GDP/capita <median value

$\beta_1 \backslash \beta_2$	0.1	0.3	0.5	0.7	0.9
0.1	0.0004	0.0286	0.2001	0.4824	0.6972
0.3	0.0002	0.0001	0.0195	0.2613	0.4777
0.5	0	0.0006	0.001	0.02	0.3325
0.7	-0.0056	-0.0019	0.0014	0.0015	0.0123
0.9	-0.0172	-0.0072	-0.0019	0.0006	0.0015

GDP/capita $\geqslant$ median value

$\beta_1 \backslash \beta_2$	0.1	0.3	0.5	0.7	0.9
0.1	0.0002	-0.0004	0.0009	0.0009	0.0141
0.3	-0.0017	0.0004	-0.0007	-0.0011	-0.0002
0.5	-0.0959	-0.004	0.0012	-0.0008	-0.0012
0.7	-0.4378	-0.1269	-0.0067	0.0014	-0.0002
0.9	-0.6745	-0.4255	-0.1203	-0.0124	0.0029

Table 3: This table documents the coefficients of linear regressions of numerical Δs as a function of the Gini coefficient $\mathcal{G}$, for the non-segregated network ($\lambda = 100$). Please refer to the caption below Tab.2 for further details. The reference value of the regression for empirical data is **-0.0018** if GDP/capita <median value and **-0.0017** otherwise.

state is more probable for higher wages). Indeed, it is difficult for low incomes to reduce what is already the bare minimum consumption.

To calibrate the model realistically as to reproduce these observations, we drop the fixed average salary $\mathbb{E}[z]$ hypothesis (which has been used in the previous sections to explore the possible scenarios of the model) and we set $\mathbb{E}^a[z] \propto \text{GDP}^a/\text{capita}$.[8]

The GDP/cap. of the United States will be used as a reference for the other countries. Without loss of generality, we fix the average wage in the US to some arbitrary value, say $\mathbb{E}[z]^{\text{US}} = 10$. Having thus fixed the value of $\mathbb{E}^a[z] = 10 \times \text{GDP}^a/\text{GDP}^{\text{US}}$ and the Gini coefficient $\mathcal{G}^a$, the value of μ^a is uniquely determined by Eq. (103).

For definiteness, we set the global sensitivity $\bar{\theta} = 4$, the global confidence level $\bar{c}_0 = 0.5$ and the amplitude of the noise to $\sigma_z = 1$, independent of a. $\bar{\theta}$ and $\bar{c}_0$ can be changed quite a bit without af-

8 Many developed countries have social policies that allow to reduce the confidence threshold via social aids of the welfare system that increase the global trust in the economy. Those policies can be modelled, for example, introducing a new parameter β_3 that modulates how $\bar{c}_0$ of a country scales with the GDP/capita setting, for example $\bar{c}_0 \to \tilde{c}_0 \mathbb{E}[z]^{\beta_3}$, where $\tilde{c}_0$ represents an arbitrary global confidence level. We have explored this extension of the model, but systematically find that $\beta_3 \approx 0$ gives the best agreement with data.

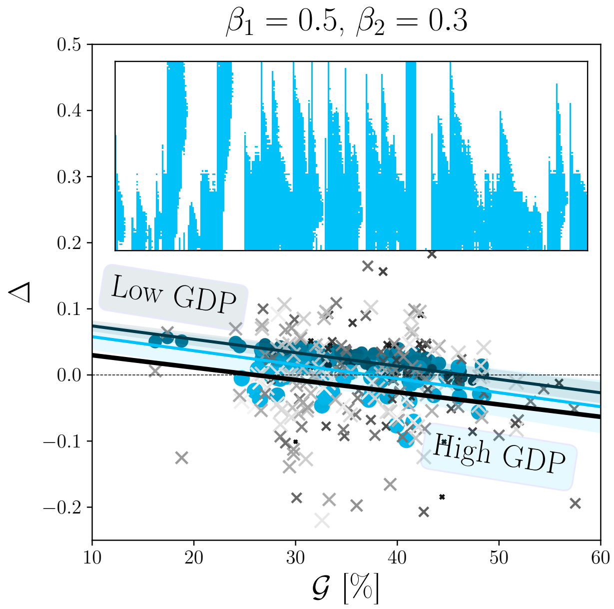

Figure 25: The panel shows the numerical simulation corresponding to the parameters that best fit real data, i.e. $\beta_1 = 0.5$, $\beta_2 = 0.3$, $\lambda = 0.01$. The brightness and the size of each point is proportional to the GDP per capita: the bigger (or brighter) the dot is, the stronger is the economy it represents. In the figure, markers ('x') correspond to real data and are shown in shades of grey. The solid black line is the linear regression through real data, which is found to be very similar for rich countries and for poor countries. The horizontal dashed black line shows the reference line $\Delta = 0$. The two coloured lines represent the linear regressions (errors on the regressions are also displayed as coloured bands) for low GDP/-cap. countries (dark blue) and high GDP/cap. countries (light blue), again very similar to one another and to real data. The upper panel shows a numerical realisation of the crisis dynamics, as in Fig. 21, for the same values of β_1, β_2, λ and with $\mathcal{G} = 0.411$ and $\mathbb{E}[z] = 10$, corresponding to the US economy in 2016 [234]. In this example, low-income households are more frequently in a low consumption state, although occasional crises also hit high earners (see left part of the time series).

fecting the quality of the final result, provided β_1 and β_2 are slightly modified as well. The value of σ_z cannot be too low (otherwise crises almost never happen) nor too high (otherwise crises are too frequent), so $\sigma_z = 1$ is a reasonable compromise. The most relevant parameters turn out to be the segregation level λ, and the exponents β_1 and β_2, which we scan but again uniformly across all countries.

We test our model for different combinations of the parameters running several simulations, each of which is based on the empirical data. Unlike the real data, where GDP/capita does not influence much Δ (the linear regression has a coefficient of -1.8×10^{-3} when

GDP/capita < median and -1.7×10^{-3} otherwise), our simulations give a linear regression that depends quite strongly on GDP/capita.

We thus split our analysis of the correlations between Δ and $\mathcal{G}$ into countries having a GDP per capita greater and smaller than the median of the points considered. The results for the numerical values of the linear regressions of the outcome of our simulations are listed in Tab. 2, together with the parameters explored.

We observe that the calibration of our model is very sensitive to the choice of β_1 and β_2, as the results differ greatly from case to case. Only for some values of the parameters do the simulations display a negative correlation between Δ and $\mathcal{G}$ independent of the level of GDP/capita. All other combinations of parameters are unrealistic and therefore discarded. We observe in particular that in the non-segregated scenario ($\lambda = 100$) there is no choice of β_1, β_2 that is compatible – even qualitatively – with empirical values.

On the other hand, when considering a segregated network (i.e. $\lambda = 0.01$), when $\beta_1 = 0.3$ and $\beta_2 \in \{0.1, 0.3\}$, or $\beta_1 = 0.5$ and $\beta_2 \in \{0.3, 0.5\}$ (highlighted in **bolt** in Tab. 2 for the segregated case and Tab. 3 for the non-segregated network) our results are consistent with empirical data, both in terms of sign and magnitude. The results corresponding to the best-case scenario is superimposed to real data in Fig. 25. (We set the brightness of each point dependent to GDP/capita: the greater, the brighter.)

The role of segregation is quite an interesting outcome of our calibration exercise. It suggests, as is intuitively plausible, that contagion effects are mostly within social classes, and less across social classes. As we noted above, our model does not properly account for financial crises, which chiefly affects the high-income class. However, a segregated network allows one to describe in an effective way the correlation in high-income consumption shocks.

A better way to model these shocks would be to allow productivity shocks ξ_t (defined in Eq. (104)) to be correlated between individuals belonging to the same social class, with a variance also depending on outcome (and therefore on countries as well, through GDP/capita). We leave this for further investigations, as one would need more microdata to calibrate such an extended model.

4.6 CONCLUSION

Let us briefly summarise what we have achieved in this chapter. First, this model extends the self-reflective DSGE framework, presented in Ch.3, to heterogeneous households, which differ in their income level and social network. Thus, consumption is also heterogeneous and is given by the Eq. (101), which appears to be novel. The trust feedback is mediated through each agent's social network, which we assume to be either only within social classes (segregated network), or between social classes (non-segregated network), with a parameter that allows us to smoothly interpolate between these two extremes. Depending on the specification of the trust feedback function, we find a rich variety of possible types of crises: propagating mainly within high-income households, or mainly within low-income households, or, in a narrow region of parameters, throughout society. Interestingly, crises are more severe for segregated networks, for which the contagion effects are stronger. Interactions between social classes tend to smooth the propagation of pessimism, because agents belonging to different social classes have different sensitivities to economic shocks. We also find that higher income inequality leads to a lower probability of global crises (holding all other parameters fixed). However, this conclusion should be taken with a grain of salt, as other effects that directly affect confidence (such as insecurity, social violence, etc.) are not considered in the model - although there is room to extend the model in that direction as well.

Finally, we compared the model's prediction with actual data, which quantify the relative decline in consumption of the lowest income decile relative to the highest income decile during recessions. Perhaps counter-intuitively, we find that in more unequal countries (with high Gini coefficients), the consumption of households with the lowest incomes tends to fall less than that of the highest incomes. This trend is mainly driven by the Gini coefficient and not by the country's GDP per capita. The model here discussed can be calibrated to reproduce this empirical result - in fact, only a small region of the parameters is compatible with the sign of the empirical effect. In particular, we find that the segregated network hypothesis is strongly favoured by the data, although other mechanisms, such as financial market fluctuations affecting only high-income households, may lead to similar effects.

Although we stick to the basic principles of standard macroeconomic models, it is useful to discuss the strengths and weaknesses of our approach with respect to recent heterogeneous extensions of TANK/HANK models, see Kaplan et al. [106]. There are (leaving aside the feedback that was introduced in the previous chapter) three main aspects of innovation. First, this model can be easily calibrated to the income inequalities in each country. These are an input to the model and, as we have seen, have a strong influence on the results. Second, in this model, it is easy to define a social structure through the coupling matrix J_{ij}. This ingredient is completely absent in TANK/HANK approaches. Last, the fact that the simultaneous presence of feedback and social interactions leads to self-amplified trust collapse. This aspect is also neglected in TANK/HANK models. In Ch.1 I mentioned how in the Ising model, the introduction of heterogeneities (the RFIM theory) or random fields and couplings can completely modify the physics of the system. Similarly, in this chapter we illustrated how the presence of heterogeneities has a key impact on the results of the model, when they are compared to the homogeneous case. It seems natural to conclude that heterogeneities play a fundamental role and cannot be neglected in macroeconomic analysis. In particular, we have obtained extremely varied and interesting results in a setup with $M = 200$ agents. The increase in the number of agents makes the social inequality curve smoother (the differences between one agent and another are proportionally smaller) but the results remain unchanged.

On the other hand, our heterogeneities are static (low-skilled workers do not become high-skilled workers, and the social network is "frozen"), while in HANK models earnings are dynamic variables as agents self-insure against possible wage losses in the future. In fact, one should expect a mixture of the two: both static and dynamically generated heterogeneities are likely to be present in the population. In any case, the model presented here is extremely versatile and can describe a variety of realistic scenarios for the formation of the crisis and its propagation through different social classes. This model is extremely ductile and prone to modification. Many possible extensions are mentioned in the text, but other directions remain to be explored. One idea would be to introduce simple laws regulating the average value of each worker's wage, depending, for example, on the output levels of the firm. In a period of crisis the productivity of some agents changes and this should, realistically, affect (downwards) their wages. This would have clear consequences for the social network, which would have to be constantly readjusted and re-adapted to the new economy. In this setup, the Gini index would vary with time, and it would be interesting to understand how. Intuitively, we think that the Gini index would grow during recessions. In such scenario, the role of politics would therefore be crucial. The lowering of wages, in fact, would lead to a downward spiral of unemployment, making the entire economy collapse within few periods. It would be crucial to implement some sort of social policies designed to restore an ideal level of employment. If the framework proposed here – which allows mixing together income inequality and trust feedbacks mediated by heterogeneous social networks – could be welded with other approaches, such as HANK models for example, it would improve our understanding of economic downturns and their impact on different strata of society.

Crisis Propagation in a Heterogeneous Self-Reflexive DSGE Model: Highlights

This chapter presents the heterogeneous extension of the model presented in the previous chapter. Unlike the TANK/HANK models, here agents are *ex ante* all different and assigned with different skills levels, affecting the wage level of agents. In this chapter skills are fixed during the entire simulations but, in future extensions of this setup, one might consider rules allowing for skills' re-distribution according to individual performances. Consumption is itself found to be heterogeneous, which is new. The other novelty introduced is the presence of a social network. Here we model interactions between agents that can be segregated and non-segregated[a]. a parameter allows one to switch smoothly from one level to another. Depending on the specification of the confidence feedback function, we find a rich variety of possible crises types: propagating mostly within high-income households, or mostly within low-income households, or else, in a narrow parameter region, across the whole society. We find that crises are more severe for segregated networks, for which contagion effects are stronger. Inter social class interactions tend to blunt the propagation of pessimism because agents belonging to different social classes have different sensitivities to economic shocks. We also find that *ceteris paribus* more income inequalities lead to a smaller probability of global crises. Last, we have compared the prediction of the model with real data, that quantify the relative drop of consumption of the lowest income decile vs. the highest income decile during recessions. Perhaps counter-intuitively, we find in more unequal countries (with high Gini coefficients), the consumption of the lowest income households tend to drop less than that of the highest incomes.

a The dynamics of segregated population has been firstly studied by Schelling, see Refs. [235, 236], but also by recent studies as Refs.[237, 238]

5

INVESTMENT ALLOCATIONS AND CAPITAL SHORTAGE IN A RBC MODEL

The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it is usual in a complex system to regard as the "causa causans" that factor which is most prone to sudden and wide fluctuation. More comprehensively, aggregate output depends on the propensity to hoard, on the policy of the monetary authority as it affects the quantity of money, on the state of confidence concerning the prospective yield of capital assets, on the propensity to spend and on the social factors which influence the level of the money wage.

— **John Maynard Keynes** [239]

5.1 INTRODUCTION

The aim of this chapter is to significantly extend our benchmark model presented in Ch.3 by including capital investment as a factor determining the trajectory of the economy. Here, we assume that capital and labour are essentially unsubstitutable, and we posit a behavioural rule for investment that takes into account both consumer confidence and the quality of returns generated by risky capital investment. Investment are therefore driven by that animal spirit already observed by Keynes (see Ch.2). This framework allows investigating the joint dynamics of confidence, capital availability and production. In a nutshell, our model tries to capture many of the ideas so clearly expressed by Keynes in the opening quotation above, while drawing largely on standard models of the business cycle. In fact, this model cannot be judged as a DSGE model because it lacks several elements, such as utility maximisation over an infinite time horizon. Some attempts to implement feedback loops in DSGE models with capital were made, but these have proved too robust. The key issue that we have struggled to solve is, among others, the explication of expectations which, in the presence of multiple equilibria, is non-trivial.

This behavioural model aims to describe a Real Business Cycle in a more realistic manner. During the Global Financial Crisis (GFC), triggered by the Lehman Brothers bankruptcy, there was a sudden collapse in confidence among both households and investors. This translated into an almost immediate collapse in both investment and consumption. These stylised facts can be observed are collected and shown in Fig.8, Ch.2. After the collapse in 2008, it took six years to recover previous confidence levels, even though investment and consumption grew in the medium term. The data speak for themselves, and it is difficult today to pin the causes of the GFC on a large (albeit persistent) exogenous shock. Rather, Keynes's story, quoted above, which attributes the rapidity of the crisis to a change in investment decisions is much more plausible. Indeed, anecdotal evidence reported by prominent actors at the time strongly suggests that the confidence collapse played an essential role in the unfolding of the crisis – see the compelling account by Ben Bernanke et al. in [240].

5.2 A BEHAVIOURAL BUSINESS CYCLE MODEL

The framework presented here hybridises some standard assumptions used in the DSGE models (see [241]) with plausible behavioural assumptions about consumption propensity and investment strategies. The environment is based on two blocks: the representative consumer and the representative firm. We do not model inflation dynamics and monetary policy, although these features should be central in a future extension of this model. Nonetheless, the phenomenology of our model is already quite rich and needs to be streamlined before exploring further the dynamics of prices.

5.2.1 *The Representative Household*

In this chapter, the feedback appearing in the household utility function in Ch.3 is implemented in a different yet equivalent way. Akin to the previous chapters the representative household, at each time period t, maximises its instantaneous utility given by

$$U_t(c_t, n_t) := G_t \cdot \log c_t - \gamma \cdot n_t^2 \,, \tag{115}$$

where c_t and n_t denote respectively the level of aggregate consumption and the aggregate amount of working hours the household provides to the firm, G_t is the (time dependent) propensity to consume out of income, and γ is the disutility of labour (which we fix to 1 for the numerical analysis).

In this model, capital takes over, through returns, in the household's budget equation. The standard DSGE formulation account for capital in the utility function of the representative household, see Sec.2.3 for references, as it owns the firm. To avoid redundancies, here we have adopted another choice. We ignore the capital's "direct" effects on utility, as the "indirect" contributions are already accounted for through consumption (this will become clearer in the next part of

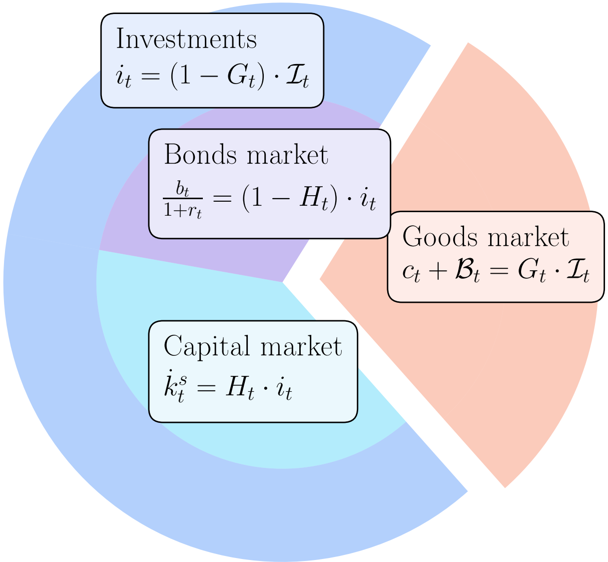

Figure 26: A schematic representation illustrating the division of income, i.e. the budget constraint, by the household.

the chapter). At each period, the household faces a budget constraint given by its real income $\mathcal{I}_t$,

$$\mathcal{I}_t := \omega_t \cdot n_t + \frac{b_{t-1}}{1+\pi_t} + q_{t-1} \cdot \frac{k_{t-1}}{1+\pi_t} , \tag{116}$$

which is funded by three sources: (i) the real wage rate ω_t paid by the firm for a unit of labour n_t, (ii) the real value of the maturating single-period bonds b_{t-1}, purchased at the time $t-1$ at the price $(1+r_{t-1})^{-1}$ and paying $(1+\pi_t)^{-1}$ at time t, where r_t is the interest rate and π_t is the inflation rate, and (iii) the realised yield q_{t-1} per unit of real capital k_t that the firm pays to the household in return for investment. We henceforth assume a constant interest rate $r = 0.15\%$ and inflation $\pi = 0.1\%$, keeping in mind a unit timescale corresponding to a month or quarter.

Total spending, correspondingly, consists in good consumption (with the price of goods set to unity), purchases of new bonds and topping up the firm's capital. Maximisation of the household's utility (Eq. (115)) leads to the familiar state equation

$$n_t \cdot c_t - \frac{G_t \cdot \omega_t}{2\gamma} = 0 , \tag{117}$$

describing the trade-off between consumption and labour in the current period t.

Interestingly, Eq. (117) can also be interpreted in a way that lends itself to a natural generalisation for investment decisions. Suppose one starts with a *time independent* utility function, Eq. (115) with $G_t \equiv 1$, which is now optimised under the constraint that the total budget devoted to consumption is a *fixed* fraction $G_t \in [0, 1]$ of the income $\mathcal{I}_t$, i.e.

$$c_t = G_t \cdot \mathcal{I}_t. \tag{118}$$

It is easy to show that the very same equation Eq. (117) immediately follows. We posit that the remaining fraction $1 - G_t$ of income is invested in bonds and capital, i.e.

$$i_t = (1 - G_t) \cdot \mathcal{I}_t \,. \tag{119}$$

A fraction of the financial portfolio, say $H_t \cdot i_t$ (with $H_t \in [0, 1]$), is allocated to productive capital, and the remainder $(1 - H_t) \cdot i_t$ is invested in bonds – see Fig. 26 for a pie chart summarising the household spending and investment decision.

The capital level available to the firm thus evolves as

$$k_t = (1 - \delta) \cdot k_{t-1} + H_t \cdot (1 - G_t) \cdot \mathcal{I}_t \,, \tag{120}$$

where δ is the capital depreciation rate. The remaining investment is allocated to bonds at price $(1 + r)^{-1}$, so

$$\frac{b_t}{1 + r} = (1 - H_t) \cdot (1 - G_t) \cdot \mathcal{I}_t \tag{121}$$

The quantities G_t and H_t aim to capture confidence effects and the attractiveness of risky capital investment, respectively, and are specified in section 5.2.5 below.

5.2.2 *The Representative Firm*

The economy's productive sector is made up of a single representative firm, which transforms labour n_t and capital k_t into a composite good y_t consumed by the representative household. The firm's production technology is given by a Constant Elasticity of Substitution (CES) function with constant returns to scale,[1]

$$y_t = z_t \cdot \left(\alpha \cdot k_t^{-\rho} + (1 - \alpha) \cdot n_t^{-\rho}\right)^{-\frac{1}{\rho}} \,, \tag{122}$$

where $\alpha = 1/3$ is the capital share in production, $1/(1 + \rho)$ is the elasticity of substitution between capital and labour with $\rho > 0$, and $z_t > 0$ is a stationary exogenous technological process. It is given by $z_t = \bar{z} e^{\xi_t}$, where ξ_t follows an AR(1) process:

$$\xi_t = \eta_z \cdot \xi_{t-1} + \sqrt{1 - \eta_z^2} \cdot \mathcal{N}(0, \sigma_z^2) \,, \tag{123}$$

1 In full generality, the CES function should be written as $\left(\alpha \cdot k_t^{-\rho} + (1 - \alpha) \cdot (\kappa n_t)^{-\rho}\right)^{-\frac{1}{\rho}}$, where κ is another parameter. However, one can always set $\kappa = 1$ at the expense of rescaling the disutility of labour parameter according to $\gamma \to \kappa\gamma$.

with first-order autocorrelation η_z, which affects the correlation time of the technology shocks. (In the following we will fix $\eta_z = 0.5$, corresponding to a correlation time of a few months). The base level $\bar{z}$ corresponds to the most probable value of productivity. Note, importantly, that $\bar{z}$ has units of $[\text{Time}]^{-1}$, i.e. the amount of goods that can be produced *per unit time* for a given level of capital and labour. As our focus is on economic fluctuations, we abstract from production growth in the present model, i.e. the secular dependence of $\bar{z}$ on time.

The CES production function nests two important limits that affect economic dynamics. As $\rho \to 0^+$, the production function becomes perfectly elastic and recovers the Cobb-Douglas form ($y_t^{CD} = z_t n_t^{1-\alpha} k_t^{\alpha}$), whereas in the limit $\rho \to +\infty$ the firm produces via an inelastic Leontief function ($y_t^L = z_t \min(n_t, k_t)$).[2] Throughout the following, we choose $\rho = 7$, corresponding to a near Leontief limit, i.e. a very small amount of substitutability between capital and labour. We will briefly comment in section 5.3.7 the impact of higher substitutability.

The firm maximises its target profit $\mathbb{P}_t$

$$\mathbb{P}_t = p_t \cdot y_t - \omega_t \cdot n_t - q_t^* \cdot k_t \ , \quad (p_t \equiv 1), \tag{124}$$

with respect to the labour supply n_t and the capital level k_t, where p_t is set to unity and correspondingly ω_t is the real wage and q_t^* is the real rent on capital. Under the assumption that the market clears, i.e.

$$y_t = c_t \ , \tag{125}$$

one finds

$$\tilde{w}_t = (1-\alpha)\left(\frac{\tilde{c}_t}{n_t}\right)^{1+\rho} \tag{126}$$

$$\tilde{q}_t^* = \alpha\left(\frac{\tilde{c}_t}{k_t}\right)^{1+\rho} \tag{127}$$

where, generically, $\tilde{x} := x/z$. Note that, as it should be, wages, consumption and rent on capital are all in units of z, i.e. unit timescale (e.g. a month or a quarter).

Combining the household's state equation, Eq. (117) and the equation for the real wage, Eq. (126), the consumption level c_t must satisfy

$$\tilde{c}_t^2 = \frac{G_t}{2\gamma}(1-\alpha)^{\frac{2}{\rho}}\left[1-\alpha\left(\frac{\tilde{c}_t}{k_t}\right)^{\rho}\right]^{1+\frac{2}{\rho}} \ , \tag{128}$$

As both sides of Eq. (128) are monotonous, this ensures a unique solution for any given level of capital k_t and consumption rate G_t. As expected, the consumption at time t increases if the capital k_t is increased and/or the consumption rate G_t is increased.

2 Keeping the parameter κ free (see previous footnote), the Leontief function would read $y_t^L = z_t \min(\kappa n_t, k_t)$, i.e. κ^{-1} measures the amount of labour equivalent to one unit of capital.

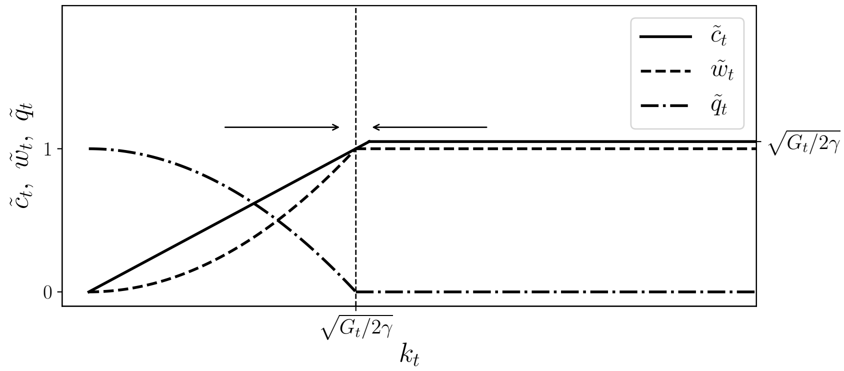

Figure 27: The figure shows the behaviour of rescaled consumption $\tilde{c}_t$, wages $\tilde{w}_t$, and rent on capital $\tilde{q}_t^*$ as a function of k_t, in the Leontief limit, i.e. $\rho \to +\infty$. Note the qualitative change of behaviour between $k_t > \sqrt{G_t/2\gamma}$ and $k_t < \sqrt{G_t/2\gamma}$.

5.2.3 *The Leontief Limit*

In this section we discuss in detail the Leontief limit of the equations derived in the previous section. Such an analysis will greatly help to understand the dynamics of the model that will be described below.

5.2.3.1 *Abundant Capital*

Assume first that $\tilde{c}_t < k_t$ and $\rho \to \infty$. Then $(\tilde{c}_t/k_t)^\rho \to 0$ and one finds

$$\tilde{c}_t \approx \sqrt{\frac{G_t}{2\gamma}} \tag{129}$$

This is only consistent with our working hypothesis when

$$\frac{G_t}{2\gamma} < k_t^2. \tag{130}$$

In this regime one finds, using Equation (117):

$$n_t = \tilde{c}_t \tilde{w}_t, \tag{131}$$

which once plugged back in Equation (126) leads to

$$\tilde{w}_t = (1-\alpha)^{1/(2+\rho)} \approx 1. \tag{132}$$

Since $\tilde{c}_t < k_t$, one concludes from Eq. (127) that the rent on capital $\tilde{q}_t^*$ is exponentially small. Intuitively, as labour is the limiting factor, consumption is directly proportional to how much the household chooses to work and the productivity at that time, while capital has no distinct effects on the economy.

5.2.3.2 *Scarce Capital*

Now let us look at the regime:

$$\frac{G_t}{2\gamma} > k_t^2. \tag{133}$$

We introduce the notation $\beta_t := 2k_t^2\gamma/G_t$ for further use. We hypothesise that the solution for $\tilde{c}_t$ in this regime is of the form

$$\tilde{c}_t = k_t e^{-x_t/\rho} \tag{134}$$

where $x_t = O(1)$ is to be determined. Plugging in Eq. (128), we find:

$$k_t^2 e^{-2x_t/\rho} = \frac{G_t}{2\gamma}(1-\alpha)^{-2/\rho}\left(1-\alpha e^{-x_t}\right)^{1+2/\rho}, \tag{135}$$

or, for $\rho \to +\infty$,

$$e^{-x_t} = \frac{1-\beta_t}{\alpha}. \tag{136}$$

The equation for $\tilde{q}_t^*$ then leads to

$$\tilde{q}_t^* = \alpha e^{-x_t} = 1 - \beta_t, \tag{137}$$

which indeed vanishes when $\beta_t = 1$, correctly matching the regime where capital is plentiful, whereas $\tilde{q}_t^*$ tends to unity when $\beta_t \to 0$, i.e. where $k_t \to 0$.

With $\tilde{c}_t = k_t e^{-x_t/\rho}$, one finds from Equation (117)

$$n_t = \frac{G_t\omega_t}{2\gamma k_t} e^{x_t/\rho}. \tag{138}$$

Finally, plugging into the equation for wages,

$$\tilde{w}_t^{2+\rho} = (1-\alpha)e^{-x_t}\beta_t^{1+\rho}, \tag{139}$$

or in the limit $\rho \to \infty$,

$$\tilde{w}_t = \beta_t + O(\rho^{-1}), \tag{140}$$

and hence $n_t = k_t + O(\rho^{-1})$. Again, this solution matches with the result $\tilde{w}_t = 1$ obtained for $\beta_t \geqslant 1$.

5.2.3.3 *Discussion*

A summary sketch of the above results is provided in Fig. 27, as available capital k_t is varied. Part of the dynamical properties of our model can be inferred from this figure: when capital is lush, return on capital is small and investment decreases (i.e. H_t decreases). If investment falls below the level of capital depreciation δ, then k_t will fall until the level $\sqrt{G_t/2\gamma}$ is reached. At this point, return on capital q_t^* increases, promoting investment. When consumption propensity G_t increases, k_t may fall behind, again leading to an increase of q_t^*. Hence, we expect a regime where the economy stabilises close to the point where $k_t \approx \sqrt{G_t/2\gamma}$, where capital and labour are tracking each other, and interest on capital and wages are neither very small, nor saturated to their maximum value $w_{max} = q_{max}^* = z$.

5.2.4 *The Risk of Investment*

Rigidity and costs to capital usage are typically introduced through adjustment costs to capital utilisation (e.g. see [242]). In this chapter we take a different route. Rather than empowering the household to choose the firm's utilisation rate, we suppose the household invests in capital and gives operational control of capital to the firm. In exchange, it is promised a return q^* per unit capital and per unit timescale (month or quarter). However, as the volatility of the stock-market attests to, such a return is not assured. Hence, we introduce an intrinsic state-dependent risk to the returns on capital[3] $\xi \in [0, 1]$ as a modifier, such that the rate actually paid by the firm is:

$$q_t = q_t^* \cdot \epsilon_t \leqslant q_t^* \, . \tag{141}$$

The random variable ϵ is distributed as

$$p(\epsilon) = a \cdot \epsilon^{a-1}, \tag{142}$$

where parameter a controls the intensity of the risk. Note indeed that $\mathbb{E}[\epsilon] = a/(1+a)$ and $\mathbb{V}[\epsilon] = a/(2+a)(1+a)^2$. Hence, the larger the value of a, the more $p(\epsilon)$ is concentrated around $\epsilon = 1$ (full payment). This formulation of risk implies that the representative firm pays out at most the marginal productivity of capital, but more likely only pays a fraction of this, corresponding to an effective description of financial distress and bankruptcy within a representative firm setup. In most simulations, we set $a = 15$, such that the return is on average 93.75% of the promised return. In an extended version of the model, the parameter a could itself be a function of the state of the economy (in particular of the availability of capital), as a form of additional feedback.

5.2.5 *Spending and Investing*

The model laid out in Sections 5.2.1-5.2.4 contains two dynamic variables, the consumption rate G_t in Eq. (118) and the investment allocation rate H_t in Eq. (119), which have not been specified yet. These two variables are responsible for the feedback mechanisms which are at the core of the dynamical evolution of our model economy. Here we elaborate on these mechanisms and provide the economic intuition behind them.

5.2.5.1 *The Consumption Rate*

As in Refs. [156, 243], we postulate that the consumption rate G_t (or propensity, see section 5.2.1) is a function of the *consumer confidence*

3 More sophisticated distributions can be considered. We use this simple form to keep the number of parameters of the model as small as possible.

index $\mathcal{C}_t$, that we model as a real variable $\in [-1, 1]$ and, possibly, on the difference between the expected inflation rate $\widehat{\pi}_t := \mathbb{E}_t[\pi]$ and the bond rate r_t:

$$G_t := G_t(\mathcal{C}_t, \widehat{\pi}_t - r_t, \dots), \tag{143}$$

where the dependence on the second variable is a way to effectively encode the content of the standard Euler equation without explicitly introducing an intertemporal optimisation of utility, and where the ... leave room to possible additional variables. But since in this part of the manuscript we assume both inflation and interest rates to be constant, the second variable will be dropped altogether. As far as the first variable is concerned, we follow the intuition developed in Ch.3 , where we postulated that confidence of a given household is impacted by the level of consumption of *other* households in the previous time step. In a mean-field limit, this self-reflexive mechanism writes

$$\mathcal{C}_t = \tanh\left(\theta_c \cdot (c_{t-1} - c_0)\right), \tag{144}$$

The parameter c_0 is a "confidence threshold" and $\theta_c > 0$ sets the width of the consumption interval over which the transition from low confidence to high confidence takes place. One could introduce, as in e.g. [244–246], the impact of macroeconomic news as an extra contribution to the argument of the tanh function. This would describe how the consumer confidence index is further modulated by some exogenous shocks.

Back to the consumption rate G_t, we write

$$G_t \quad = \quad \frac{1}{2}\left[G_{\min} + G_{\max} + (G_{\max} - G_{\min}) \cdot \mathcal{C}_t\right] \tag{145}$$

where $0 \leqslant G_{\min} < G_{\max} \leqslant 1$ are the minimum and maximum proportions of income the household will consume [4] We fix $G_{\min} = 0.05$ to ensure the household will consume whenever its income is positive (necessary consumption). Similarly, we set $G_{\max} = 0.95$ to account for a minimal form of precautionary savings in response to some uncertainty regarding the future.

The intuition behind Eqs. (144) and (145) is discussed in details in Ch.3, and we will therefore only recall the key aspects:

- when consumption is above the threshold, $c > c_0$, there is high confidence in the future of the economy, and the household maintains high consumption with a minimal amount of savings to sustain capital levels, i.e. $G_t \to G_{\max}$.

4 To avoid any confusion, it is key to distinguish the differences between the consumption feedbacks (functions that were taken to be tanh) present in this thesis. In Ch.3, $G_t \propto F_t^{1/3}$ represented the consumption propensity, and therefore it was unbounded. In Ch.4, F_t was a measure of the work provided by the household to the firm (as in Ch.3). Here G_t represent a portion of the total income the representative household allocates in the good market. Therefore, in this chapter, G_t must be positive and bounded by 1.

- when consumption is below the threshold, $c < c_0$, the consumption rate collapses, inducing the household to save more for the future, and $G_t \to G_{min}$.
- The parameter θ_c modulates the households' reaction to a breach of necessary consumption. For high θ_c, the household requires only a relatively small shock below c_0 to reduce the consumption rate to its minimum. Conversely, as $\theta_c \to 0$ the household becomes unresponsive to the state of the economy, consuming half its income regardless of high or low preceding consumption. [5]

Following the phase diagram obtained in Ch.3, there are four distinct "phases", i.e. regions of qualitatively comparable dynamics, that are distinguished by the bi-stability of G_t. We can observe in particular a phase of high persistent consumption with no crises (phase A), high consumption with short downward spikes (phase B^+), or a phase with alternating periods of high consumption (booms) and low consumption (busts), i.e. phase C.

5.2.5.2 *The Investment Allocation*

In each period, the household must allocate its savings between one-period bonds and capital. It does so through an allocation decision H_t based on the household's observation of the economy, and its beliefs about future risk and return. The novelty of our model lies in the behavioural foundation that determines the proportion of new investment dedicated to bonds, H_t.

Investment beliefs are shaped by two factors: (i) an estimate of the expected risk-adjusted excess returns to capital investment, given by a Sharpe ratio $\mathcal{S}_t$ [247], and (ii) the current confidence level $\mathcal{C}_t$ about the future state of the economy.

The Sharpe ratio $\mathcal{S}_t$ is an estimate of the risk-adjusted real return, $q_t - \delta$, of investing capital in the firm versus holding risk-free bonds (b_t) paying r_t. It increases as the returns to capital increase or become less volatile. We assume that estimates of the future Sharpe ratio are only based on exponential moving averages of past (observable) realised returns, which is a form of extrapolative beliefs.[6] In other words, i.e. the mean μ^q and standard deviation σ^q of the return stream are computed as

$$\mu_t^q = \lambda \cdot \mu_{t-1}^q + (1-\lambda) \cdot q_t \tag{146}$$

$$(\sigma_t^q)^2 = \lambda \cdot (\sigma_{t-1}^q)^2 + (1-\lambda) \cdot (q_t - \mu_t^q)^2 \tag{147}$$

$$\mathcal{S}_t := \mathcal{N} \cdot \frac{\mu_t^q - r_t - \delta}{\sigma_t^q} \tag{148}$$

5 The intermediate levels of θ_c describe the smoothness of the adjustment to consumption shocks.

6 See [248, 249] for recent empirical work on extrapolative beliefs.

with an exponential moving average defined by a gain parameter $\lambda \in (0, 1)$, corresponding to a memory timescale equal to $\mathcal{T}_\lambda := 1/|\log \lambda|$: a larger λ implies that a higher weight is given to recent observations. The factor $\mathcal{N} \approx 1/4$ is quite arbitrary, but chosen such that, when compared to the confidence in Eq. (149) below, the two terms are of similar magnitude. (Note that this choice is, in fact, immaterial, since changing $\mathcal{N}$ is equivalent to rescaling the parameter ν defined in Eq. (149) below.)

The interpretation of the Sharpe ratio is as follows: a positive signal $\mathcal{S}_t > 0$ suggests that the expected real return to capital investment exceeds the returns to risk-free bonds. The magnitude of $\mathcal{S}_t$ is inversely proportional to the risk of capital investment, as measured by the estimated volatility σ_t^q. Thus, in a high-volatility environment, the signal might be positive but weak.

The second indicator potentially influencing the household investment decision is the confidence index, $\mathcal{C}_t$, as previously defined. In periods where the household has low confidence, there is a reduced impetus to invest in risky assets because households wish to guarantee next-period income. These are often periods of crisis with a higher volatility in returns. Since bonds are risk-free, this leads to a higher allocation of funds to bonds, *ceteris paribus*. Conversely, higher confidence about the future means more appetite for risk, and hence a higher fraction of the savings invested in the capital of firms and a lower fraction invested in bonds.

We postulate that the propensity, H_t, to make risky bets is a function of the overall *sentiment* Σ_t, computed as a linear combination of the Sharpe ratio and of the confidence:

$$\Sigma_t = \nu \cdot \mathcal{S}_t + (1 - \nu) \cdot \mathcal{C}_t, \tag{149}$$

where $\nu \in [0, 1]$ is the weight the household gives to its estimates of risk-adjusted return $\mathcal{S}_t$ and its confidence level $\mathcal{C}_t$. When $\nu = 1$, the household's confidence plays no role in the investment rule. For positive Sharpe ratio and confidence indicator, the sentiment is positive, $\Sigma_t > 0$, indicating a willingness to invest in risky capital. But if $\nu < 1$ sentiment can turn negative even when the Sharpe ratio is high, because of a high level of anxiety about the future state of the economy, encoded as a negative value of $\mathcal{C}_t$.

Finally, the unbounded sentiment Σ_t is transformed into a portfolio allocation to capital $H_t \in [0, 1]$ via,

$$H_t = \frac{1}{2}\left[H_{max} + H_{min} + (H_{max} - H_{min}) \cdot \tanh(\theta_k \cdot \Sigma_t)\right], \tag{150}$$

where H_{max} and H_{min} represent the maximum and the minimum proportion of total investment i_t invested into capital. In the following, the allocation decision is bounded between $H_{min} = 0$ and $H_{max} = 1$, which precludes any divestment (or short-selling) of capital or bonds.[7] The parameter θ_k represents the sensitivity of the portfolio allocation to the agent's sentiment and sets the width of the sentiment interval over which the capital allocation goes from H_{min} to H_{max}, that is how *polar* the investment decision is. For $\theta_k \to \infty$, the allocation becomes binary, leading to either $H_t = H_{min}$ when sentiment is negative or $H_t = H_{max}$ when sentiment is positive.

In the following part, we fix the sensitivities to a rather high value $\theta_k = \theta_c = 15$, such that the transitions between different regimes are sharp.

5.2.6 *Summary and Orders of Magnitude*

In this section we have set up a business cycle model incorporating two behavioural mechanisms: a self-reflexive consumption rate decision, very similar to the one introduced in Ch.3, and an investment allocation decision. The novelty of this part lies in the behavioural foundation that determines the proportion of new risky investment H_t, which depends directly on three key parameters, λ, ν, θ_k, describing the "sentiment" of the household, i.e. its risk aversion. H_t also indirectly depends on the risk intensity parameter a and the capital depreciation rate δ. The consumption decision depends on two parameters c_0 and θ_c that define the household confidence about its future welfare.

In the following, we discuss how the parameters of these two feedback mechanisms strongly affect the model's dynamics. Note that a very important parameter of the model is the baseline productivity $\bar{z}$, which fixes the scale of the consumption, wages and rent on capital (all per unit timescale). In the following, we choose $\bar{z} = 0.05$, corresponding to an annual productivity of capital of 20% if the unit time step is a quarter and 60% if it is a month.[8]

Among all the parameters of the model, three have an interpretation in terms of time scales:

- η_z, which appears in the dynamics of the productivity shocks, that we have fixed to 0.5 throughout this study, corresponding to a timescale $\mathcal{T}_\eta = 1/|\log \eta_z|$ of a few months ;
- λ, which is the gain parameter used by investors to estimate the Sharpe ratio of risky investments, corresponding to a timescale $\mathcal{T}_\lambda = 1/|\log \lambda|$. Our default value will be $\lambda = 0.95$, corresponding to $\mathcal{T}_\lambda \approx 20$ or 5 years if the unit time is a quarter or a month respectively;

7 One could allow for divestment by $H_{min} < 0$, however, this would require a more elaborate form for Eq. (150).

8 To estimate an appropriate order of magnitude for $\bar{z}$, we considered the gross value added by non-financial corporations in the U.S. divided by the current cost net stock of fixed assets together with total wages (as a proxy for labour), which shows a downward trend to approximately 28% p.a.

- δ, the capital depreciation rate, which we choose in the range 0.001 – 0.02, corresponding to a typical replacement time of capital $\mathcal{T}_\delta = 1/|\log(1-\delta)| \approx 12$ – 250 years when the unit time is a quarter, and three times less if it is a month. Hence, $\delta = 0.001$ means essentially no depreciation of capital.

The role and the effect of varying these timescales is studied in detail in Section 5.3.7. An important remark, at this stage is that, while our choice of one quarter as the unit time step is quite arbitrary, a combination of parameters that is crucial for the properties of the model is the dimensionless product $\bar{z} \cdot \mathcal{T}_\delta \approx \bar{z}/\delta$, i.e. how many goods can be produced (per unit capital) over the life-cycle of capital.

5.3 CRISES AND PHASE DIAGRAMS

In this section we first investigate numerically the phase diagram of our self-reflexive business cycle model and highlight the different dynamical features that the model can generate. We choose as control parameters those which govern the behaviour of our two feedback mechanisms: the consumption propensity G_t and the risky investment decision H_t. In order to navigate through the following paragraphs, let us explain in a nutshell what is expected to happen in the model.

If a productivity shock causes confidence to drop, consumption propensity G_t and consumption both drop as well, whereas the saving rate $1 - G_t$ increases. Because consumption drops, unemployment rises and capital becomes superfluous, leading to a decrease in the rent on capital q^*. Because the fraction of savings invested in capital H_t depends both on q^* (through the Sharpe ratio) and on the level of confidence (with a weight $1 - \nu$), the amount invested in risky capital, given by $(1 - G_t) \cdot H_t \cdot \mathcal{I}_t$, can either increase (if the factor $1 - G_t$ dominates) or decrease (if the factor H_t dominates), depending on parameters and conditions. In the second situation, and if capital depreciation is fast, one may face a situation where consumption is impaired and capital becomes scarce at the same time, making recovery more difficult and leading to long periods where the economy is trapped in a low output state.

5.3.1 *Crises Indicators*

We focus on two distinct phenomena exhibited by our model: consumption crises and capital scarcity.

- Consumption crises occur in periods where the household's consumption, c_t, falls below its threshold, c_0. In other words, we have a low demand for consumption which leads the econ-

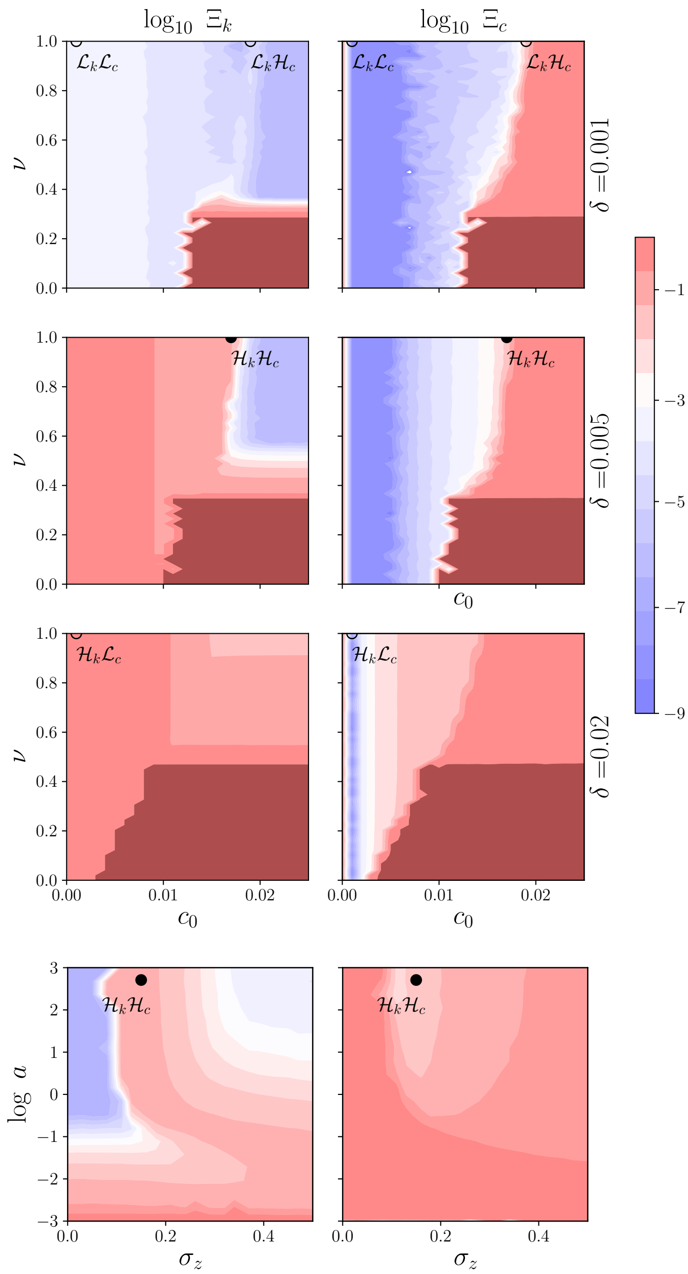

Figure 28: This figure shows different sections of the phase space. The left-most panels display the $\log_{10}$ probability of capital shortage Ξ_k as a function of the Sharpe ratio weight ν and of the confidence threshold c_0. The right column displays the $\log_{10}$ probability of consumption crises Ξ_c as a function of the same parameters. The dark red zones correspond to the regions where $\Xi_{k/c} > 0.99$, i.e. crises are permanent. Each column corresponds to a different choice of the capital depreciation, from $\delta = 0.001$ to $\delta = 0.02$. We set $a = 15$ and $\sigma_z = 0.15$. As δ increases, the $(\mathcal{H}_k, \mathcal{H}_c)$ regimes becomes widespread. In each panel, we have marked the points chosen to illustrate the different phases of the model, together with their label (see text). The dynamics trajectories and the corresponding histograms are reported in Fig. 29. The two bottom panels show another section of the phase space, varying $\log\ a$ and σ_z, with $c_0 = 0.017$, $\nu = 1$ and $\delta = 0.005$ fixed (the two solid dots there correspond to the same solid dots of the middle panels, in the $\mathcal{H}_k\mathcal{H}_c$ phase).

omy into a stagnating low-output state. The severity of such consumption crises is measured as

$$\Xi_c = \frac{1}{T}\sum_{t=0}^{T}\left(1-\frac{c_t}{c_0}\right)\Theta(c_0-c_t), \tag{151}$$

where $\Theta(x \geqslant 0) = 1$ and $\Theta(x < 0) = 0$ and T is the total simulation time. This indicator counts the fraction of time consumption c_t is low, weighted by the relative distance between c_t and c_0.

- Since we are considering an economy defined by low substitutability between capital and labour (i.e. $\rho \gg 1$ in the CES production function), we define capital scarcity as the periods where production is determined by capital levels, i.e. $k_t \leqslant n_t$. The severity of capital crises is similarly measured as:

$$\Xi_k = \frac{1}{T}\sum_{t=0}^{T}\left(1-\frac{k_t}{n_t}\right)\Theta(n_t-k_t). \tag{152}$$

In a sense, one can consider these two phenomena as demand and supply crises.

- In the consumption crisis state, the household does not wish to spend on consumption. Hence, we see a low aggregate demand. Provided capital depreciation is low, this is also a state of excess capital ($k > n$) and low returns on capital.
- In the capital scarcity state, the firm is bound in its production by the supply of capital. Hence, it can be viewed as a form of supply crisis.

Both phenomena can be more or less frequent, and at first glance unrelated, but closer scrutiny reveals that in some regions of parameters, these two types of crises interact with one another. To differentiate between characteristic behaviours, we distinguish between four different phases in the space of the parameters defined by the values that the indicators Ξ_k and Ξ_c take: $(\mathcal{L}_k, \mathcal{L}_c)$, $(\mathcal{L}_k, \mathcal{H}_c)$, $(\mathcal{H}_k, \mathcal{L}_c)$, $(\mathcal{H}_k, \mathcal{H}_c)$, where $\mathcal{L}$ and $\mathcal{H}$ represent the "low prevalence" and "high prevalence" of each phenomenon c or k, respectively. There is however no strict definition of the boundary between high and low prevalence regimes. As a convention, we consider that the crisis prevalence is high when $\Xi \sim 10^{-2}$.

Given this setup, we first focus on the effects of three key parameters: the depreciation rate δ, the weight ν of the Sharpe ratio in the investment decision, and the consumption threshold c_0. Other parameters are fixed to $\bar{z} = 0.05$, $\lambda = 0.95$ (i.e. $\mathcal{T}_\lambda = 20$), $\alpha = 15$ and $\sigma_z = 0.15$ [9].

9 An interesting extension of the current model is to allow the parameter α describing the default risk on capital to depend on the state of the economy, since defaults are more frequent when the economy is in a low output and low investment regime.

Figure 28 presents heat-maps of the severity of capital crises $\log_{10} \Xi_k$ (top) and consumption crises $\log_{10} \Xi_c$ (bottom) across parameter combinations, where red indicates high prevalence. We show two representative sections of the parameter space: the planes (c_0, ν) (left) and (σ_z, a) (right). Since we present the logarithms of $\Xi_{k,c}$, the crossovers between high and low prevalence of different phases are rapid but smooth, i.e. there are no sharp phase transitions that characterise the system's behaviour. From each phase, we study a point of the line $\nu = 1$ and different values of c_0 (marked by points in Figure 28), with all other parameters fixed and plot the dynamics of consumption c_t, labour n_t, capital k_t, and the measured Sharpe ratio $\mathcal{S}_t$ in Figure 29. Note that changing the value of parameters (including ν) while staying in the same phase leads to qualitatively similar trajectories.

5.3.2 *Prosperous Stability*

As shown in Figure 29, upper row, the $\mathcal{L}_k\mathcal{L}_c$ phase is characterised by a stable capital surplus, low interest on capital and rare consumption crises. The depreciation of capital δ is so small that even with a puny level of investment, capital is always in excess and labour is the limiting factor. The stable capital surplus, in combination with a low confidence threshold c_0, means that productivity shocks z_t hardly ever reach the required magnitude to trigger a consumption crisis, and if it does, recovery is almost immediate.

The corresponding bottom panel of Figure 29, shows that the consumption level has normal fluctuations, entirely due to exogenous productivity shocks z_t, around a single high-consumption equilibrium. A corollary of the large capital excess is that the labour supply is nearly constant (extremely narrow-distribution in the $\mathcal{L}_k\mathcal{L}_c$ panel of Figure 29).

As the depreciation rate δ increases, the average excess of capital supply over labour shrinks, increasing the prevalence of capital scarcity. Accordingly, the phase $\mathcal{L}_k$ quickly disappears upon increasing δ, leading to a pervasive $\mathcal{H}_k\mathcal{L}_c$ phase (see e.g. Fig. 28, third column, which shows that capital is always scarce when $\delta = 0.02$). As δ is further increased, the $\mathcal{L}_c$ phase is more and more confined to small values of c_0, i.e. when confidence is intrinsically robust.

5.3.3 *Prevalent Capital Scarcity*

The $\mathcal{H}_k\mathcal{L}_c$ phase, second row of Fig.29, is characterised by persistent capital scarcity with rare consumption crises, and is confined within a low c_0 "band" in the (c_0, ν) plane when δ is large enough. Since c_0 is low, confidence of is generally high and therefore the household systematically consumes a large proportion G_t of its income, leaving only a small share for investment. Because of capital depreciation,

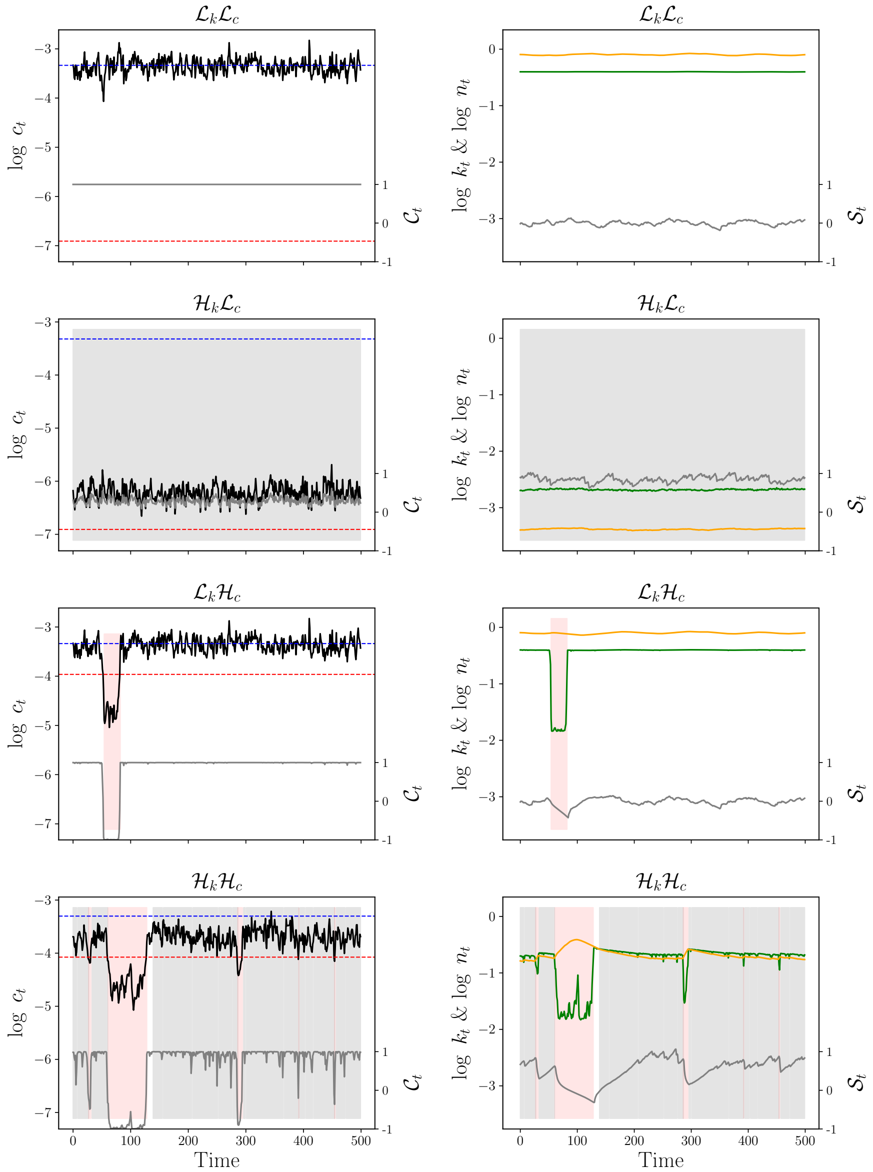

Figure 29: The panels show sample dynamics trajectories for each phase presented in Fig. 28 (marked by circles). In the left column, the solid black line corresponds to the consumption c_t, while the dashed red and blue horizontal lines show, respectively, the confidence threshold c_0 and the average consumption in the $\delta = 0$ scenario. Grey (resp. pink) background indicate capital scarcity (resp. consumption crisis). The right column presents the dynamics of capital k_t (solid orange), labour n_t (solid green) and Sharpe ratio $\mathcal{S}_t$ (solid grey, with levels shown on the right y-axis). For all simulations $\nu = 1$, $a = 15$, $\sigma_z = 0.15$. Specific parameters are $\mathcal{L}_k\mathcal{L}_c$: $\delta = 0.001$, $c_0 = 0.001$, $\mathcal{L}_k\mathcal{H}_c$: $\delta = 0.001$, $c_0 = 0.019$, $\mathcal{H}_k\mathcal{L}_c$: $\delta = 0.02$, $c_0 = 0.001$, $\mathcal{H}_k\mathcal{H}_c$: $\delta = 0.005$, $c_0 = 0.017$.

the economy settles in a regime where $k_t < \sqrt{G_t/2\gamma}$, meaning that production is limited by capital, wages are low and rent on capital is high (i.e. the left region in Fig. 27). Hence, the average consumption

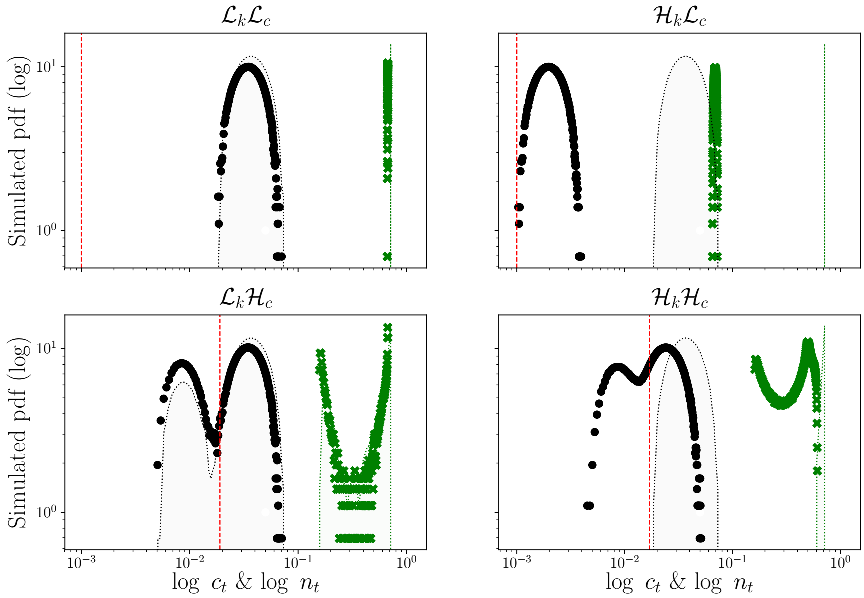

Figure 30: The set of graphs shows the histograms of consumption (black dots) and labour (green crosses), referred to the dynamics presented in Fig.29, in a log − log scale. The green (resp. grey) dashed curve corresponds to the $\delta = 0$ baseline value for labour (resp. consumption), with c_0 indicated as a vertical red line. In the $\mathcal{H}_c$ phase, the histograms of consumption and labour become bi-modal, corresponding to high output and low output regimes. For all simulations $\nu = 1$, $a = 15$, $\sigma_z = 0.15$. Specific parameters are listed in the caption of Fig.29 .

level is lower than the maximal consumption level reached in the $\mathcal{L}_k\mathcal{L}_c$ phase – see Figure 29, second column.

But since c_t is now closer to c_0, consumption crises are lurking around and the economy can flip into the $\mathcal{H}_k\mathcal{H}_c$ if c_0 increases and/or if productivity shocks are stronger (higher σ_z). This is clearly confirmed by the phase diagram of Fig. 28. In fact, comparing the phase diagrams for $\delta = 0.005$ and $\delta = 0.02$, we see that faster depreciation of capital converts large swaths of $\mathcal{H}_k\mathcal{L}_c$ phase into $\mathcal{H}_k\mathcal{H}_c$. Hence, in this case, investment crises (i.e. the supply side) do trigger consumption crises (i.e. the demand side) by reducing the difference between c_t and c_0 – see also the discussion in section 5.3.5.

5.3.4 *Prevalent Consumption Crisis*

When the depreciation rate is sufficiently small but the confidence threshold increases, capital remains abundant but self-reflexive confidence crises can hurl the system into a low consumption, low employment regime as a result of random productivity shocks. This is the $\mathcal{L}_k\mathcal{H}_c$ phase. Since capital is high, its level does not impact the level of production, and interest on capital is small. Hence, the model becomes completely equivalent, in this regime, to the one studied in Ch.3, where the dynamics is characterised entirely by the consumption propensity G_t and is dominated by frequent consumption crises, induced by breakdown of collective confidence.

The dynamics, displayed in the third row of Figure 29, are akin to the ones obtained in the C phase of Ch.3, Fig.14. In fact, consumption then displays bi-stable dynamics, where high and low consumption regimes alternate. Correspondingly, the distributions of consumption and labour reveal a secondary peak centred around the low-consumption equilibrium.

Note that during consumption crises (i.e. $G_t \searrow$) capital becomes even more abundant relative to labour (recall that one needs to compare k_t with $\sqrt{G_t/2\gamma}$) and therefore return on capital and Sharpe ratio both fall, as can be seen in the pink shaded region of Fig. 29, third column. If we are in a region where consumption crises are short enough compared to both the time $\mathcal{T}_\lambda$ over which the Sharpe ratio is estimated and the capital depreciation time $\mathcal{T}_\delta$, then one can avoid a capital crisis when confidence comes back. Otherwise, the economy enters a turbulent $\mathcal{H}_k\mathcal{H}_c$ phase with both capital and consumption crises.

5.3.5 *Capital and Consumption Crises*

This final $\mathcal{H}_k\mathcal{H}_c$ phase, bottom row of Fig.29, has both persistent capital scarcity and consumption crises. As anticipated above, capital crises can trigger consumption crises, because capital scarcity drives consumption closer to the confidence threshold c_0, below which consumption drops and precautionary savings increase. One can then enter a doom loop (similar to Keynes' famous paradox of thrift) where now capital is too high and leads to a reduction of incentive to invest away from bonds. Hence, as shown in the fourth column of Fig. 29, capital and labour fluctuate around low levels, with intertwined periods of capital scarcity (grey regions) and high unemployment (pink regions). The Sharpe ratio gyrates rather strongly between negative values and values close to unity, with a significant negative skewness. The economy is unstable and always far from its optimal state.

Recall that we have fixed the interest rate on bonds to a constant value. But with a massive demand for bonds, as expected in the $\mathcal{H}_k\mathcal{H}_c$ phase, one should expect the government to borrow at low rates and prop up the economy with public investment, a feature not modelled in the current framework, but certainly worth accounting in future extensions.

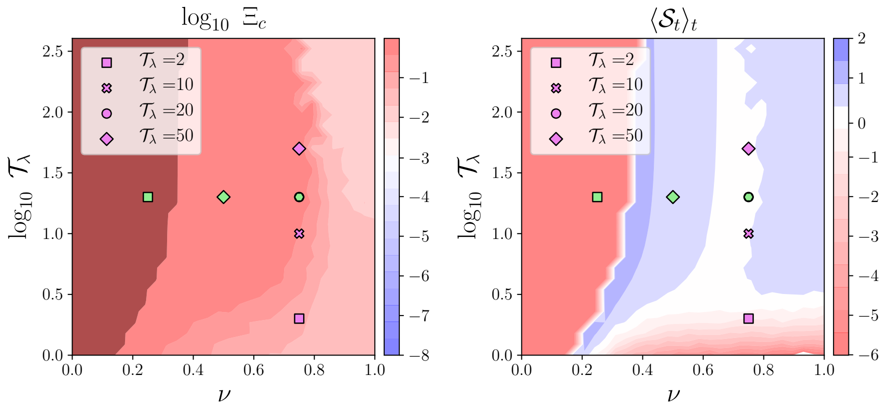

Figure 31: The left and right heat-map shows $\log_{10} \Xi_c$ (resp. average Sharpe ratio) as a function of the memory timescale $\mathcal{T}_\lambda$ and the weight parameter ν. The dark red zones correspond to the regions where $\Xi_c > 0.99$, where crises are permanent.

5.3.6 *Summary*

To summarise, we have identified four qualitatively different phases of the dynamics. Possibly the most interesting (and novel) one is $\mathcal{H}_k\mathcal{H}_c$, where capital scarcity is persistent, thereby triggering consumption crises. In this phase the economy is unstable, as capital becomes scarce the likelihood of a consumption crisis increases, and vice versa, low consumption drives rent on capital down and increases the risk aversion of investors.

We have underlined the role of the capital depreciation rate δ in determining the fate of our model economy. In fact, when capital and infrastructure are sufficiently long-lived such that $\bar{z} \cdot \mathcal{T}_\delta$ is large, the economy reaches a stable and prosperous state $\mathcal{L}_k\mathcal{L}_c$, provided self-induced confidence crises are rare enough (i.e. c_0 small). Conversely, when $\bar{z} \cdot \mathcal{T}_\delta$ is low, capital depreciates too quickly and this dents the rents that can be expected by investors. The economy quickly becomes under-capitalised and inefficient, especially because the dearth of capital makes confidence crises more probable, paving the way for the existence of a dysfunctional $\mathcal{H}_k\mathcal{H}_c$ region in the phase diagram.

5.3.7 *Investment and Crisis Recovery*

In the previous section, we have explained how capital depreciation can cause instabilities, and the appearance of a $\mathcal{H}_k\mathcal{H}_c$ phase where both capital and consumption undergo regular crises. In this section, we want to explore the influence of the memory timescale $\mathcal{T}_\lambda$, which is the history span over which investors assess the Sharpe ratio of capital investment, and of the sentiment parameter ν on the time needed for recovery when in a crisis period. We focus on this turbulent phase

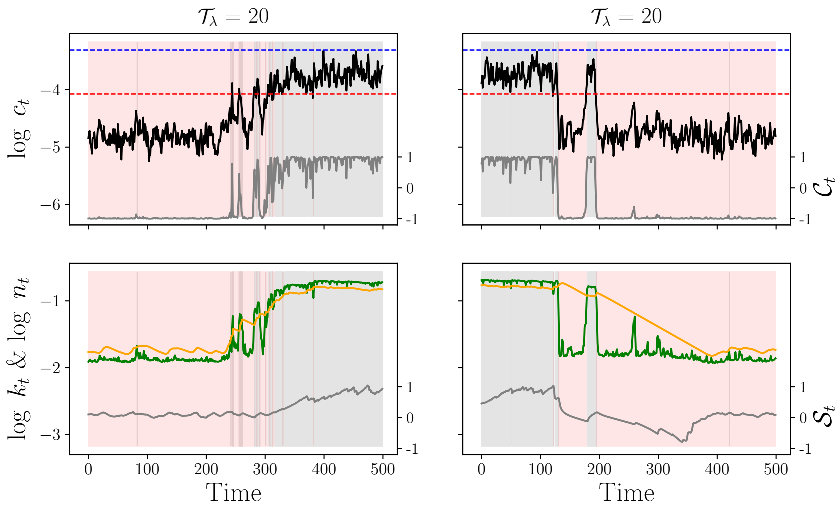

Figure 32: The panels show snapshots of the dynamics of c_t, k_t and n_t, corresponding to recovery (left) and crisis formation (right), both for $\mathcal{T}_\lambda = 20$ and $\nu = 0.75$.

of the economy. We will fix the other two timescales $\mathcal{T}_\delta, \mathcal{T}_\eta$ defined in section 5.2.6 to, respectively, 200 and 2.

Our benchmark will thus be the $\mathcal{H}_k\mathcal{H}_c$ point in Fig. 28, corresponding to $\lambda = 0.95$, $\delta = 0.005$, $\nu = 1$ and $c_0 = 0.017$ (with α and σ_z also fixed at their baseline values). Looking at the statistics of the high consumption periods and of the low consumption periods, we conclude that the prosperous periods last a time $\mathcal{T}_>$ of the order of $\mathcal{T}_\delta = 200$ (data not shown), whereas crises are rather short, of the order of $\mathcal{T}_< \approx 10$, see Fig. 33, plain vertical lines in the third graph of the bottom row, which corresponds to $\mathcal{T}_\lambda = 20$, i.e. $\lambda = 0.95$. The full distribution of $\mathcal{T}_<$ and of the Sharpe ratio $\mathcal{S}$ for $\nu = 1$ are shown in light grey, and reveals that whereas its time averaged value of $\mathcal{S}$ is clearly positive and equal to $\langle \mathcal{S} \rangle_t \approx 0.71$, its full distribution is uni-modal but quite broad and negatively skewed.

Fig. 33, left graphs shows the consumption crisis prevalence Ξ_c and the average Sharpe ratio $\mathcal{S}$ as $1-\nu$ (weighing confidence in the investment allocation decision) and $\mathcal{T}_\lambda$ are varied. One sees that decreasing ν or increasing $\mathcal{T}_\lambda$ leads to an increase of Ξ_c, at least in the range shown, $\mathcal{T}_\lambda \sim 300$. The evolution of the average Sharpe ratio is more complex, reflecting the non-trivial shape of its distribution function (bi-modal and skewed, see below). But certainly as agents pay less attention to the actual return on capital and are more affected by the level of confidence, the average Sharpe becomes strongly negative (red region of the diagram) and the economy gets trapped forever in a low consumption, low investment regime where $\theta_k\Sigma$ is negative (see Eq. (150)).

Now, let us look at a cut along the direction $\nu = 0.75$, corresponding to a 25% weight given to confidence in the allocation decision, as $\mathcal{T}_\lambda$ is varied. For this particular value of ν, the average Sharpe ratio is close to zero and only weakly depends on $\mathcal{T}_\lambda$ (Fig. 33, bottom left graph). But from the bottom row of Fig. 33, we see that when $\mathcal{T}_\lambda \sim 10$, the distribution of Sharpe ratios becomes *bi-modal* and with a skewness that decreases as $\mathcal{T}_\lambda$ increases. This can be rationalised as follows:

- The peak corresponding to positive Sharpe ratios comes from prosperous periods, where consumption is high and capital relatively scarce, leading to a positive return on capital q^*: see Fig. 33, top right graphs: in the high consumption phase, the orange line (capital) is below the green line (labour).
- The peak corresponding to zero Sharpe comes from crises periods, where capital is in slight excess of labour, leading to a small return on capital (see again Fig. 33, top right graphs, and section 5.2.3).
- The fat left tail corresponding to negative Sharpe ratios comes from the transitory periods between high confidence and low confidence, when consumption and labour collapse but capital depreciates much more slowly. In this case, return on capital plummets and the Sharpe ratio becomes negative.
- As $\mathcal{T}_\lambda$ increases, the weight of these transitory regimes in the estimate of the Sharpe ratio becomes small, and the fat left tail disappears, as crises become less frequent and much longer.[10]

Whereas the length of the prosperous periods $\mathcal{T}_>$ is unchanged compared to the benchmark $\nu = 1$ case for all values of $\mathcal{T}_\lambda \sim 10$, the length of the crisis periods $\mathcal{T}_<$ increases by more than 100 times as ν is decreased from 1 to 0.75 (compare the grey line and the coloured points in the bottom row in Fig. 33). The first observation is due to the fact that the Sharpe ratio estimated when in a high output period is clearly in positive territory and quite insensitive to $\mathcal{T}_\lambda$ (see the histograms in Fig. 33). This means that capital supply is also independent of $\mathcal{T}_\lambda$ and that the confidence collapse mechanism must be identical to the one described in Ch.3 and not triggered by a lack of investment. [11]

On the contrary, the mechanism by which confidence is *restored* is strongly impacted by the value of the memory time $\mathcal{T}_\lambda$. When the economy is in a consumption crisis, the returns on capital are very small. Thus, averaged over a sufficiently long time period, the

10 For very large $\mathcal{T}_\lambda$, the situation changes again, see below.

11 This is not to say that the crisis frequency is not related to capital abundance. As already noted in section 5.3.5, as capital depreciation δ increases, available capital decreases, which leads to a lowering of output c_t. Since the distance between c_t and the threshold c_0 is a crucial determinant of the probability of a confidence crisis, the region $\mathcal{H}_k\mathcal{H}_c$ becomes pervasive as δ increases, see again Fig. 28.

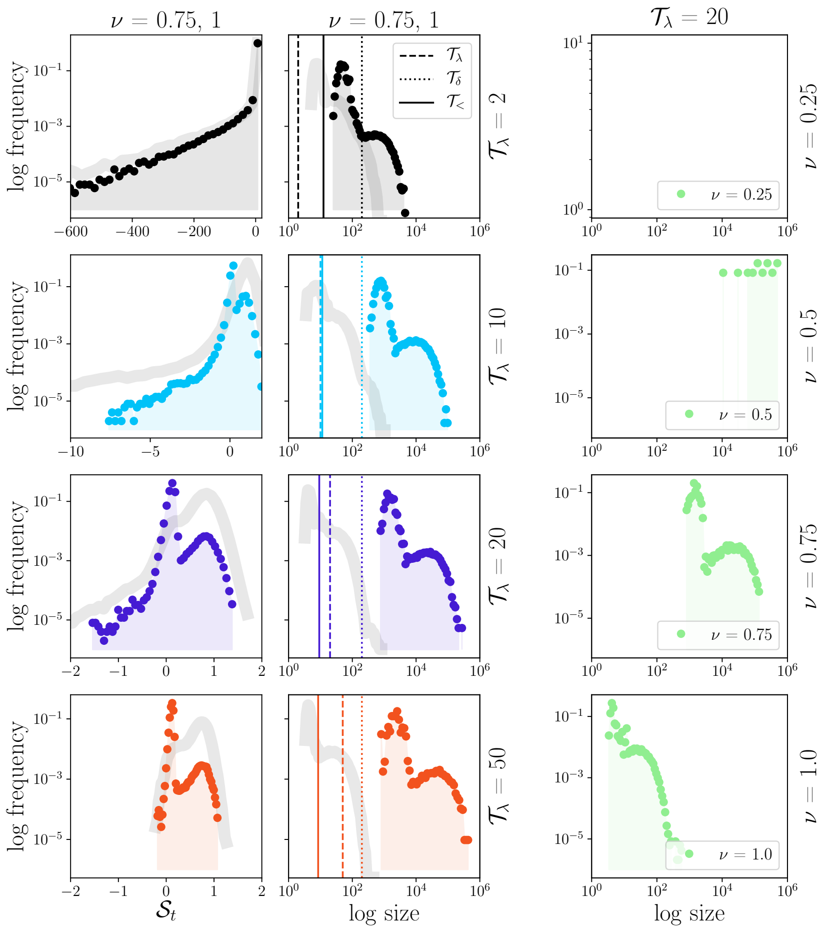

Figure 33: The two left columns show the histograms of Sharpe ratio $\mathcal{S}$ and crisis duration $\mathcal{T}_<$ for four values of $\mathcal{T}_\lambda$: 2, 10, 20 and 50, all for the same value of ν (shown as symbols in the two heat-maps on the left). The faded grey lines show the same histograms in the benchmark case $\nu = 1$ that correspond to the $\mathcal{H}_k\mathcal{H}_c$ point in Fig. 28. Other parameters used are: $\delta = 0.005$, $c_0 = 0.017$, $\sigma_z = 0.15$ and $\alpha = 15$.

Sharpe ratio is well-defined and also small (see the narrow peaks in the Sharpe ratio distribution in Figure 33). Combined with the low confidence dampener on sentiment $(1-\nu) \cdot \mathcal{C}_t$, this leads to a negligible investment flow. So whereas positive productivity shocks should put the economy back on an even keel, the level of capital is lagging, which creates a ceiling that prevents consumption (and hence confidence) from increasing substantially and returning to the high consumption case.

Interestingly, the dependence of $\mathcal{T}_<$ on $\mathcal{T}_\lambda$ is in fact non-monotonic. For very large $\mathcal{T}_\lambda \sim 1000$, the memory of prosperous periods persists even during the crises, so that the Sharpe ratio and investment always remain high. In such cases, $\mathcal{T}_<$ abruptly drops back to small values

~ 5 (data not shown). With minimal probability, however, the system remains trapped in a crisis forever.

In the opposite case of a small enough $\mathcal{T}_\lambda$, the short periods where consumption increases due to productivity shocks allow sufficiently rapid increases in capital rent to encourage immediate investment. This is enough to prop up capital and allows confidence to be fully restored as labour and consumption will grow with the limiting factor k_t. For an example of these positive spikes of consumption, see top centre panel in Fig. 33. The same effect takes place if ν is increased back to 1, where only realised Sharpe affects investment. In this case, the drag on capital due to low confidence levels is absent, and the system can pull itself out of the rut much more efficiently, leading to shorter crisis periods. But for lower values of ν (higher impact of household confidence on the investment propensity), the dearth of capital in crisis periods is such that the economy is unable to ever recover, i.e. $\mathcal{T}_< = +\infty$ for all purposes.

From a policy point of view, reducing interest rates has the direct effect of increasing the Sharpe ratio and reducing the return to bonds, thus promoting investment and making the transition back to the high consumption state easier. However, this may require the central bank to set interest rates r to negative values, as r, which might already be close to zero due to prior crises. Besides monetary policy, other measures that improve confidence (e.g. central bank messaging) and/or promote investment into productive capital would have a similar impact (for instance if the government decides on strong fiscal measures that include investment into productive capital, such as through mission-oriented policies or infrastructure spending).

Finally, let us mention that while the existence of consumption crisis is independent of the substitutability parameter ρ, the duration of the low investment, low consumption periods is also highly sensitive to substitutability effects. We have indeed found that when ρ is sufficiently small, i.e. for production functions closer to Cobb-Douglas than to Leontief, recovery is much faster. This could have been expected: lack of capital can now be compensated by labour, expediting the transition back to a prosperous state of affairs.

5.4 FROM COBB-DOUGLAS TO LEONTIEF

The choice of production function plays a key role in the phenomenology of this model.

In the first attempts we made to build up this model, we assumed a Cobb Douglas production function, as it is usually the case in a DSGE literature. However, the Cobb Douglas framework “decoupled” the effects of the two feedbacks. The solution was achieved once we assumed the firm’s production was modulated by the Constant Elasticity of Substitution (CES) function, as in Eq.(122). The parameter

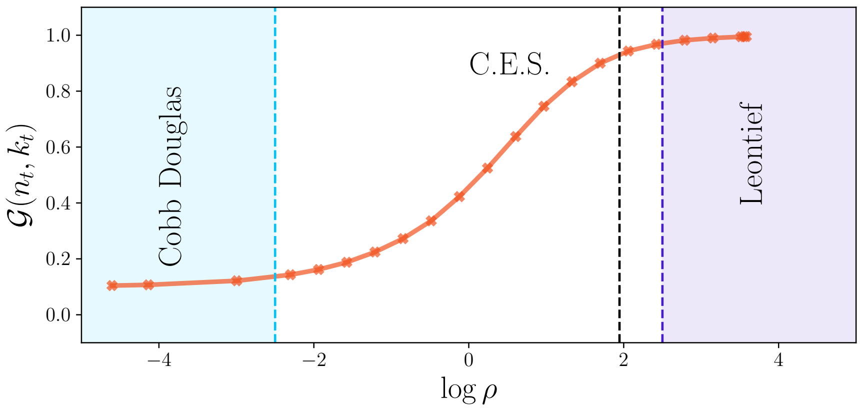

Figure 34: The correlation function $\mathcal{G}(n_t, k_t)$ is displayed as a function of $\log \rho$. Within the light blue/violet regions we recover the Cobb Douglas / Leontief limit respectively. The vertical dashed black line is the reference for $\rho = 7$, that has been used across the chapter. Other parameters used are: $\nu = 1$, $\lambda = 0.95$, $\delta = 0.005$, $c_0 = 0.017$, $\sigma_z = 0.15$ and $\alpha = 15$.

ρ when varied allows to shift the elasticity of substitution between capital and labour.

In all this chapter we have considered $\rho = 7$. We recall that for $\rho \to 0$ capital and labour are perfectly substitutable as the production function becomes Cobb Douglas, while in the limit $\rho \to \infty$, the firm's output, as expressed in Eq.(122), reads $y_t = z_t \min(k_t, n_t)$ and capital and labour are perfectly unsubstitutable.

In Sec.5.2.3 we have discussed the limit $\rho \to \infty$. When the capital is scarce, it becomes fully correlated to labour as, following the reasoning of Sec.5.2.3, $n_t = k_t + O(\rho^{-1})$ while the correlation should vanish in the Cobb Douglas limit. Fig.34[12] reflects this idea and shows the progressive changing in the correlation function between the trajectories of n_t and k_t restricted to periods where the capital is scarce. The graph allows distinguishing the two limits quite precisely: for values of $\rho \ll 1$ the correlation is ~ 0.1, i.e. the two variables are almost independents. Increasing the exponent ρ smoothly changes this relation up to the point where correlations become ~ 1 for values of $\rho \sim 20$. What is then the effects on the trajectories?

We set our-self in the turbulent regime $\mathcal{H}_k\mathcal{H}_c$, and we let ρ vary. The results are shown on Fig.35. Three stylised facts emerges: First, as discussed above, as ρ increases and the capital is scarce the solution found in Sec.5.2.3 holds and, see bottom panel of Fig.35, the two trajectories almost superpose. *Vice versa*, when $\rho = 0.01$ the two variables are independent.

12 Because of machine errors, the real values of ρ one can explore via numerical simulations is bounded by $\rho \in [0.05, 35]$. however, $\rho = 0.01$ is small enough to illustrate the Cobb Douglas regime and vice versa $\rho = 35$ shows the Leontief limit.

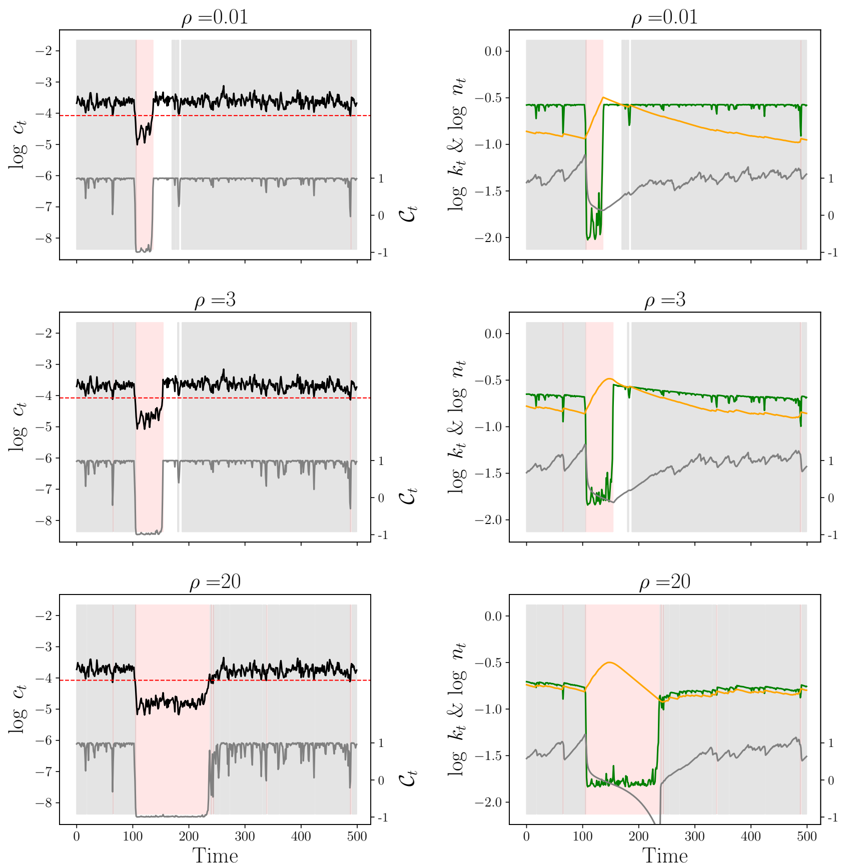

Figure 35: The panels show snapshots of the dynamics of c_t, k_t and n_t, for different values of ρ. $\rho = 0.01$ essentially corresponds to the Cobb-Douglas limit, while $\rho = 20$ illustrate the Leontief scenario. Other parameters used are: $\nu = 1$, $\lambda = 0.95$, $\delta = 0.005$, $c_0 = 0.017$, $\sigma_z = 0.15$ and $a = 15$.

Second, with the increase of ρ crises are more persistent. This is explained by a coordination of two effects. On the one hand, as goods market clear, when $\rho \gg 1$ the consumption is limited by the scarcity of labour. Following the intuition, in order for the panic effects to be reabsorbed, a larger (upwards) productivity shock is needed. In the scenario $\rho \ll 1$, the output is boosted by the abundance of capital (see Fig.35). Therefore, the economy needs a rather smaller relative exogenous shock to recover. Consequently, jumps from low $\rightarrow$ high consumption states are more likely, as discussed in Sec.3.4.

The second actor is the Sharpe ratio. The persistence of the recessions allows, for $\rho \gg 1$, the Sharpe ratio to degrades (given the same λ). This fact is reflected by the histograms of the Sharpe ratio's evolution for the three cases shown in Fig.33. When $\rho = 20$ the Sharpe becomes negatively skewed and this further amplifies the economical recessions compared to the Cobb-Douglas case. Thus, we believe that the results obtained in Sec.5.3.7 are a characteristic of the CES production function and the value of ρ we chose.

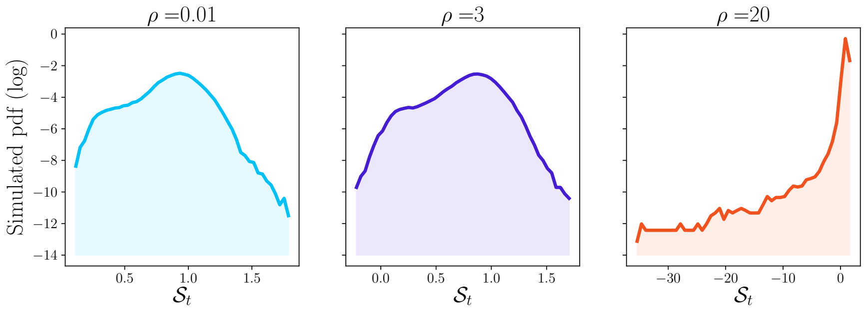

Figure 36: The graph shows the histogram (log scale) of the Sharpe ratio for the three dynamics displayed on Fig.33.

5.5 CONCLUSION

In this chapter, that concludes my original work, we have extended the original formulation of the DSGE models with capital to a behavioural model of the real business cycle in which labour and capital are almost unsubstitutable. This unsubstitutability is given by a CES-type production function.

In the model presented here, consumption and investment are controlled by two feedbacks. The first one recalls (although implemented slightly differently) the self-reflective household confidence presented in chapter Ch.3. The second modulates the propensity of households to invest in the capital market and/or in the (safer) bond market. Important in this model is the role of sentiment measured by the Sharpe ratio and the confidence level itself. The role of confidence, instead, is very similar to the one presented in the previous two chapters: productivity shocks to output can lead to crises in which consumption jumps abruptly from a high to a low equilibrium level. Depending on the parameters of the model, these crises can be more or less frequent, and the periods of low consumption can be of varying lengths: short spikes (V-crises) or long, protracted phases (L-crises). The introduction of capital influences the resulting phenomenology of the economy, which, in this case, is rather rich. Indeed, capital can be abundant (in which case labour is the limiting factor of production) or scarce, in which case it will be the limiting factor of production. Compared to the models illustrated above, the introduction of investors preferences plays a fundamental role in the business cycle as:

- Higher capital depreciation rates, *ceteris paribus*, lead to capital scarcity and limit production. This makes the economy more prone to crises of confidence, increasing their prevalence;
- The increased influence of the level of confidence in capital allocation decisions creates a feedback loop similar to Keynes' paradox of thrift, destabilising and trapping the economy in a state of suboptimal low consumption;

- The time during which the economy remains in a state of low consumption is highly sensitive to the time frame in which investors calculate the Sharpe ratio. Increasing this time frame leads to slow adjustments in investment. As a result, the instantaneous increase in capital returns due to productivity gains is not sufficient to stimulate the propensity to invest. This leads to a persistence of capital scarcity and prevents the economy from breaking out of the low return trap.

Our results have several policy implications. First, as discussed in Ch.3, the presence of self-reflective feedback loops requires governments and monetary authorities to manage confidence in the future prospects of the economy. Although confidence indices are routinely measured by survey institutions (see Fig. 8 in Ch.2), the inclusion of such indices in DSGE macroeconomic models and the importance of narratives have never really been seriously considered beyond the impact of news shocks on productivity. [13]

In addition to communication and narratives, monetary authorities should also directly promote investment, particularly during recessions. This is necessary to prevent the economy from becoming trapped in an environment of low output and low confidence. Although this conclusion seems perfectly intuitive, our model reveals that a lack of capital can prolong downturns by orders of magnitude, and convert V-shaped downturns into L-shaped ones. Increasing investment in working capital can be done through traditional channels, by lowering the risk-free interest rate (possibly making it negative) or through Keynesian-style direct investment in infrastructure and innovation, which have the double effect of raising capital productivity and supporting household confidence.

There are of course many directions in which our model should be extended and improved. The first one is to allow interest rates and inflation to be dynamic variables, and to introduce an explicit monetary policy having the double role to control inflation and confidence, via narratives [226]. Important would be the inclusion of a feedback mechanism between confidence and the timescale $\mathcal{T}_\lambda$ or the sentiment parameter ν. This would allow setting potentially relevant panic effects in the model and capture more realistically what happened during the GFC in 2008, for example.

13 For example, the work of [246] which reflects interactions and complementarity. Also [250] has shown that single news shocks (confidence shocks) are sufficient to empirically adjust the effects of business cycles. See also [156].

Capital Scarcity and Self-Reflexive Confidence: Highlights

In this chapter, I present a behavioural model based on a classical DSGE. What is new compared to previous chapters is the introduction of capital and a "pseudo" financial market. The (representative) agent decides whether to allocate his resources in the bond market or in the riskier capital market. The factory will pay with a certain probability the promised returns. Another innovation is that the factory produces according to a CES function, in which capital and labour are not completely substitutable. This leads to periods when capital is scarce and consumption is limited by its level. This effect combined with the proven consumption feedback generates panic effects whose magnitude exceeds expectations. In particular, if crises are sufficiently persistent, agents lose confidence in the market and stop investing in the capital market. This generates a self-reflexive loop that leads the economy to collapse. In this model, we propose several approaches that governments should undertake to alleviate the duration of economic crises. This model comes closest to describing the events thought to have occurred during the 2008 Global Financial Crisis.

Part IV

CONCLUSION

6 CONCLUSION

If one accepts ergodicity in Economics then it will be included in the "realm of sciences" rather than in the "realm of history" which does not apply the same laws with time invariance symmetry.

— **Paul Anthony Samuelson**[67]

During my thesis, I have worked on several models whose goal was to answer fundamental questions raised in recent debate within Macroeconomics. The aim was to test whether, with simple modifications, the DSGE framework could be stretched, bridging the gap with the ABM community. In all the analysis I have conducted, both by choice and academic training, my mindset was the one of a statistical physicist.

6.1 THE DIVIDE ET IMPERA METHODOLOGY

In maths as well as in Physics, the methodology *divide et impera* is often applied to break a complicated problem into pieces, and analysing each part individually. Once all the pieces of the puzzle are solved, one can put them together to find the global solution sought[1]. It is with this spirit that I have conducted my research. Scrolling through the pages one can appreciate the increasing degree of complexity of the models I present, from the inclusion of heterogeneities to the addition of capital markets, each model has its roots in the previous one (the model presented in Ch. 3) which serves as a benchmark.

In this manuscript, I tried to bridge the gap between the two communities following this "step-by-step" philosophy.

Ch.3 represents our first formulation of the feedback economy, applied to the simplest monetary model.

Each household forms its propensity to consume, by observing what the average consumption of its neighbours was during the previous time period. This opens up the possibility that a relatively small decline (exogenous) in the overall production will lead to a collapse in confidence, inducing a sharp drop in economic activity. When increasing the strength of the feedback different regimes are crossed: from a phase (when the feedback is weak enough) where the model essentially behaves, albeit with increased volatility, similarly to the starting DSGE model, to a regime (when the feedback is strong enough) where even small technological shocks can induce persistent

1 This method must not be confused with the reductionist philosophy discussed in Ch.1

economic crises, in a *small shocks large business cycle* fashion. The self-reflexive feedback mechanism, introduced in Ch.3 addresses among the main criticisms of DSGEs: (i) the concept of general equilibrium is overcome by the presence of two distinct equilibria, generating high-/low output respectively and (ii) the exogenous origin of standard DSGE's economic crises is here self-propelled by endogenous mechanisms. This study highlights the importance of *narratives* [226] and the pathological presence of the *unknown knowns* as two major drivers of the economy. Even in such schematic setup, the results are much more realistic than the one described by the reference model described in Sec.2.2.

The intuition developed in Ch.3 paved the way for the understanding of its extensions, presented in Chs.4 and 5.

The model presented Ch.4 lays the foundations for a heterogeneous and connected society. The representative agent hypothesis is definitively abandoned, and the mean field approximation of Ch.3 is replaced by a multi-household heterogeneous landscape. Among the challenges of Ch.4 we had (i) to set a rule to determine the social network through which agents interact and (ii) to implement heterogeneities. The latter enter the model via different (and quenched) income levels, which, together with the segregation level of the social structure, led to very rich dynamics. Depending on the parameters, crises spread mostly within one social class or, alternatively, swing across the entire population. I also discussed, supported by numerical simulations, the critical value of the exogenous shock that triggers global events. The first two chapters are naturally interlinked and constitute an almost continuous work. The original setup is relaxed in the last part of this thesis: in Ch.5, I presented a radical extension of the benchmark model in Ch.3, where I focused on the addition of the capital market into the business cycle. This work required a great deal of effort and could only be achieved by abandoning the rigid DSGE paradigm. Not in its entirety, of course, but sufficiently so that the resulting model belongs to the behavioural RBC realm rather than to the DSGE realm. The introduction of capital led to extremely rich dynamics and combined the effects of supply- and demand-driven downturns[2]. Supply shortage as a driver of recessions is a consequence of the CES function, which does not allow for full substitution between labour and capital. In fact, when capital is scarce, the firm's output –and therefore the aggregate consumption– is limited by its level. Capital scarcity induces, with higher probability, confidence collapses. The panic so generated redirects agents' investment allocations to the safer bonds market. The combination of these two effects leads to more realistic scenarios, where economic crises are

2 Those are not a novelty for ABM models: in a multi-firms context, Assenza et al.[251] shows that Capital and Credit are essential ingredients of realistic economical recessions.

initially induced by a low level of capital and are prolonged by a lack of investment as a consequence of the drop in confidence. To further stress the importance of the *divide et impera* method, much of the phenomenology discussed in Ch.3 is still recognisable (if one looks for it) among a multitude of new effects. The setup presented in Ch.5 provides insights for monetary authorities, promoting direct Keynes-like investments (particularly during recessions) to prevent the economy from being trapped in a low output regime.

This last setup represents the natural conclusion of my thesis and opens up many possibilities for new research.

Future directions Much of the behaviour described in our models are based on non-linear effects – dictated by feedback mechanisms – related to the existence of several fixed points. In a framework where linearisation is forbidden, the exploitation of the Euler equation has proven itself to be a real challenge.

In that regard, the first and perhaps most urgent direction for future research is to establish a method for calculating inflation and, consequently, interest rates in a framework where the paradigm of the single general equilibrium is lost due to interactions.

The idea consists in finding some regimes where the linear approximation of the Euler equation holds, as done in Sec.3.5. Possibly, one can write the DSGE equations as a function of the output gap and the confidence level. This direction is of particular interest as it shows how to endow DSGE with possibly measurable confidence indices. An ongoing project done within the *Econophysix* research chair aims to address this question and should be the subject of forthcoming work.

The second direction, also under investigation (albeit still at an embryonic stage) within the *Econophysix* research chair, concerns the relaxation of the market clearing hypothesis. This is particularly challenging as the market clearing is one of the core and most rigid DSGEs' axioms. Even within the simplest monetary model, Sec.2.2, if the output of the firm does not match the household's demand, it is extremely complicated to find a coherent solution for the aggregate variables. A possible direction to get around this problem follows many ideas presented in the work of Dessertaine et al. [252]. To remain close to the DSGE approach, one might imagine an economy where markets are constantly out of equilibrium and both the supply and the demand sides correct their propositions according to the realised market gap. This can be achieved introducing forecasting functions modulating the behaviour of economic actors. The estimation of such predictive functions is not simple and complicates the problem considerably, adding additional terms to the utility maximisation. The process described is, however, very similar to the *tâtonemment* introduced by Walras, see Ref.[54] for references, when he first theorised the general equilibrium of markets as the progressive adaptation of supply and demand sides.

Other possible extensions, such as the addition of central bank and monetary policies (or a financial sector), are of great interest, especially in the context of Ch.5. This modification allows the policy makers to incentive investments by lowering the interest rates. This would increase the Sharpe ratio, making capital investments more appealing. The central bank can exploit forward guidance[226] to favour capital allocations, directly affecting the households' sentiment, Eq. (149), via, for instance, an additional term.

Finally, it would also be interesting to look for a "great unification" between the type of business cycle/DSGE behavioural models considered in this thesis and the heterogeneous agent-based models studied in the recent literature, which generically give rise to similar crises and bi-stable dynamics between high and low output regimes of the economy (see among many others [165, 253]).

Very interesting would be to merge our last work, see Ch.5 with the work done by Boissay et al.[254], where they present a model where banks are heterogeneous with respect to their intermediation skills, which give rise to an interbank market. There, moral hazard and asymmetric information allows for severe recessions that propagates endogenously. Being these two studies complementary (we focus on the consumers side) I do think that combining the two frameworks would give rise to an extremely realistic and complete macroeconomic model.

6.1.1 *Personal view*

The reader now expects me to take a clear position concerning the central debate of this thesis. Unfortunately, I cannot do so, but I can instead provide my perspective on the problem. On the one hand, I do not think that DSGE models should be ostracised as a large proportion of the warmest supporters of ABM thinks. Some DSGE ingredients are the condensation of observations, reflections and discussions, matured along many decades and therefore cannot be discarded on principle. In fact, DSGE models contain interesting elements and should be taken as inspiration for the development of new setups.

In contrast, the DSGEs have clearly proven to have serious "axiomatic" defects, that are too resilient to be modified. For example, the hypothesis of an infinitely rational representative agent places strong bounds on modelling, but also implicitly assumes that estimation is feasible as the future economy can be inferred from the past. This point is extremely delicate, and it assumes *ergodicity* in Economics (see below). The concept of full rationality is pathological, as it allows for inconsistencies. First, within a rational framework, moment's anchors are introduced via arbitrary quadratic laws, see Eqs.(42) and (45), which strongly clash with the absolute rationality of agents. Second: in a heterogeneous model of infinitely rational agents, would they maximise their utility (as it is), or rather the global utility of the society as a whole? Whatever one chooses, the concept of full rationality would inevitably be lost as the sum of the preferences might differ from the preference of the sums, as "more is different".

In real contexts, if pragmatic, people's behaviour contains a certain degree of rationality and foresight. We all make the most important choices by exploiting most of the available information, seeking to maximise our future well-being. On the one hand, it is therefore undeniable that future expectations play a key role in decision-making in many fields including finance, Macroeconomics, Monetary Economics, see Ref. [255]. On the other hand, expectations are both based on an imperfect analysis of collected data and are subject to non-rational phenomena (e.g. panic effects) that may in some cases dominate their measurement. Financial actors are a good example: they maximise their expected profit given their current level of capital by basing their allocations on observations and analyses of past stock market trends. Many, however, react impulsively if the market falls sharply, fuelling the downward spiral of prices.

Under these premises, agent-based models are proposed in the debate as the new paradigm to follow. They have the enormous advantage of adhering perfectly to Anderson's ideological manifesto "*More is different*" and they do not require any formal assumption of rational behaviour. Although their foundations rely on more flexible concepts than the rigid DSGEs, their progressive complications have led them to run into many of the criticisms that are also levelled at the DSGE community.

Some ABM enthusiasts have made enormous intellectual efforts to create extremely complex agent models comparable in number of individuals to real economies[3].

In my opinion, the issues of such large setups are several and collide with the spirit of this thesis. First, the large number of degrees of freedom makes it hard to systematically study (and therefore understand) the effect of each parameter. Without a phase diagram or, alternatively, some form of diagrammatic representation of the possible outcomes as a function of the relevant control parameters, this understanding cannot even be achieved. This partly justifies the negative appellation of "black-boxes" by which ABMs are often referred to.

The excessive complications make these extended models prone to strong redundancies in their formulation, as the macroeconomical intuition of their outputs is compromised. In fact, the role of many parameters might superpose, leading to similar effects and/or being

3 Austria in particular, see Poledna et al.[195]

negligible. These weaknesses are often difficult to spot and fixed, as the vast number of entries plays the role of a frosted glass, blurring the view of researchers. This methodology clashes, with the *divide et impera* scheme I advocate here.

Lastly, there might be an issue, common to the most sophisticated DSGE models, of over-fitting: any trajectory can be reproduced with a sufficiently large number of free parameters. But such overlap is merely artificial and must not be confused with forecasting power.

The reader may accuse me of being pessimistic, but I am not. In my opinion, the fundamental question that needs to be answered concerns the role that macroeconomic models must play.

6.2 THE CLIMATOLOGY PERSPECTIVE

As I briefly discussed in the foreword to this thesis, it is misleading to expect macroeconomic models to predict economic performance in the same way as weather forecasts do for meteorology. On the one hand, the vapour particle (H_2O molecule), not being a thinking entity, follows the laws of Physics, or paraphrasing Galileo Galilei: "*the great book of nature is written in mathematical characters*". The Navier-Stokes' equations and the laws of thermodynamics are well understood and verified, and nevertheless their predictions are dominated by uncertainties. The nature of weather forecast models is inherently chaotic and any error, however small, in the initial conditions spreads exponentially fast so that their predictive power is very limited. As a result, they only have an acceptable level of accuracy when looking at a maximum of two days ahead, and the increasing demand for accuracy translates into a smaller time window. If one is interested in knowing the probability of precipitation during the day, the window of certainty is as small as a few hours.

On the other hand, some basic core macroeconomic concepts are, by definition and not by mistake of economists, not easily measurable and naturally ill-defined. Economic agents act according to their intuition (or confidence) and can operate in a completely counter-intuitive way. Trust, as well as many other human feelings, are difficult to represent by precise mathematical expressions due to their vague definition. If economic agents were completely rational and behaved like particles of water, endogenously propagated economic crises would be a contradiction. Yet, they do occur, with cyclical regularity.

An additional problem lies in the temporal invariance of the laws governing the two systems, or, in other words, ergodicity. On the one hand, the laws of Physics are time-invariant: since the gravitational force $g \approx 9.81 m/s^2$ is a constant, the trajectories described by the throw of two distinct stones, given the same initial conditions, are identical. The result of such a simple experiment does not vary regardless of the time gap between the two events.

On the other hand, the parameters governing macroeconomic laws are constantly evolving in time and differ according to the economy one seeks to describe. The economic consequences of a change in interest rates might vary depending on the historical context in which this variation is applied. The absence of time invariance in Economics reflects the fact that society, is a constantly mutating entity: fashions, tastes and even political ideologies reflect this constant evolution. In fact, given the same observation, politicians might decide differently: a right-wing government will opt for laws that are diametrically opposed to the laws a left-wing government would opt for.

Even assuming that the previous arguments are flawed, and economic predictions share the same degree of accuracy as weather forecasts, reality is even more complex. The existence of the *unknown unknowns*, which, as I described in Chs. 2 and 3, add a component of radical uncertainty to the problem. Any effort, however precise, to calibrate macroeconomic variables will always be affected by inherent errors which, due to the chaotic nature of the models, defeat any of their claims to be predictive.

Thus, the goal of forecasting the economy with a model, however complicated, seems to be unattainable. Models must therefore be (at least for the time being) relieved from being quantitatively predictive.

What then should be their purpose? To close the climate analogy, the role of macroeconomic models should be closer to climatology. In the same way that climatology aims to study possible climate scenarios due to ongoing climate changes, economic models should serve to explore the set of future economic scenarios.

Policy makers would then have tools to analyse stylised facts, highlight possible feedback loops, reveal the potential importance of new mechanisms and show whether the economy is on the edge of a tipping point. According to the results of their analysis, central banks would be able to estimate which control parameters should be targeted by monetary policies in order to preemptively reverse the observed most dangerous economic trends. Following the logic developed in this thesis, tools such as forward guidance must be exploited to increase trust and consequently reduce the probability (and the persistence) of economic recessions.

To reconnect with the example I gave at the beginning of this thesis, rather than predicting the future trajectory of French GDP by observing the trajectories of some complicated model, and interpolating its value in two years' time, economic models should aim to explore possible scenarios arising from current trends in, for example, consumer confidence indices. Considering the global Covid-19 crisis and the natural trend of investor confidence in Europe, the models should predict an increase of, say, from 3% to 5%[4], in aggregate consumption over the next two years.

4 These percentages have only an illustrative purpose and are not realistic.

Instead of attempting to forecast the exact amount of rain that will fall in a certain area in the next hours, one can much more reliably predict if the next days will be rather humid or dry based on cyclonic activity, atmospheric pressure, and other relevant factors, and if a change of the weather conditions is likely to occur in the near future.

I further stress that to achieve this goal, efforts must be directed at those aspects that make models' formulation more plausible, yet keeping them as simple as possible. However, if reasonable, mathematical complications should not frighten researchers. Numerical simulations serve as a "telescope of the mind" [165], filling the analytical void through the generation of synthetic data.

To improve the understanding of existing correlations among macroeconomic variables, models must be confronted with empirical observations, as their calibration is nowadays increasingly at reach due to the large amount of data available. The cause-effect relationships observed empirically cannot be neglected, but rather must be accommodated within the models, thereby facilitating a better description of economic activities. Exploiting data, it is possible to assign to each parameter realistic values by modulating their relative magnitude. This work helps discern which region of the phase diagram is the most plausible, narrowing down the otherwise vast number of possibilities. The efforts of organisations like the OECD to analyse and collect data are commendable.

Data speak and must be listened to.

I hope that the reading of this thesis has been generally enjoyable and interesting. I also hope that I have conveyed to the reader to the reader my deep interest in the topics that have accompanied for the past three years.

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